DIFFERENTITATION

Single Correct Answer Type

1. A differentiable function $f(x)$ is defined for all $x > 0$ and satisfies $f(x^3) = 4x^4$ for all $x > 0$. The value of $f'(8)$ is

   a) $\dfrac{16}{3}$
   b) $\dfrac{32}{3}$
   c) $\dfrac{16\sqrt{2}}{3}$
   d) $\dfrac{32\sqrt{2}}{3}$

2. If $y = x - x^2$, then the derivatives of y^2 w.r.t. x^2 is

   a) $2x^2 + 3x - 1$
   b) $2x^2 - 3x + 1$
   c) $2x^2 + 3x + 1$
   d) $2x^2 - 3x - 1$

3. If $y = e^{\frac{1}{2}\log(1+\tan^2 x)}$, then $\dfrac{dy}{dx}$ is equal to

   a) $\dfrac{1}{2}\sec^2 x$
   b) $\sec^2 x$
   c) $\sec x \tan x$
   d) $e^{\frac{1}{2}\log(1+\tan^2 x)}$

4. For $|x| < 1$, let $y = 1 + x + x^2 + \cdots$ to ∞, then $\dfrac{dy}{dx} - y$ is equal to

   a) $\dfrac{x}{y}$
   b) $\dfrac{x^2}{y^2}$
   c) $\dfrac{x}{y^2}$
   d) xy^2

5. If $f(x) = x^n$, then the value of $f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + \cdots + \dfrac{(-1)^n f^n(1)}{n!}$ Is

   a) 2^n
   b) 2^{n-1}
   c) 0
   d) 1

6. $\dfrac{d^2}{dx^2}(2\cos x \cos 3x)$ is equal to

   a) $2^2(\cos 2x + 2^2 \cos 4x)$
   b) $2^2(\cos 2x - 2^2 \cos 4x)$
   c) $2^2(-\cos 2x + 2^2 \cos 4x)$
   d) $-2^2(\cos 2x + 2^2 \cos 4x)$

7. If $y = x^{x^{x^{x \cdots \infty}}}$, then $x(1 - y\log x)\dfrac{dy}{dx}$

   a) x^2
   b) y^2
   c) xy^2
   d) xy

8. If $f(x) = \dfrac{g(x)+g(-x)}{2} + \dfrac{2}{[h(x)+h(-x)]^{-1}}$ where g and h are differentiable function, then $f'(0)$ is

   a) 1
   b) $\dfrac{1}{2}$
   c) $\dfrac{3}{2}$
   d) 0

9. $\dfrac{d}{dx}\left[\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})\right]$ is equal to

   a) $\dfrac{1}{2\sqrt{x(1-x)}} - \dfrac{1}{\sqrt{1-x^2}}$
   b) $\dfrac{1}{\sqrt{1 - \left\{x\sqrt{1-x} - \sqrt{x(1-x^2)}\right\}^2}}$
   c) $\dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{2\sqrt{x(1-x)}}$
   d) $\dfrac{1}{\sqrt{x(1-x)(1-x)^2}}$

10. If $y = \sin^2 \alpha + \cos^2(\alpha + \beta) + 2\sin\alpha \sin\beta \cos(\alpha + \beta)$, then $\dfrac{d^3 y}{d\alpha^3}$, is

    a) $\dfrac{\sin^3(\alpha + \beta)}{\cos\alpha}$
    b) $\cos(\alpha + 3\beta)$
    c) 0
    d) None of these

11. If $z = \log(\tan x + \tan y)$, then $(\sin 2x)\dfrac{\partial z}{\partial x} + (\sin 2y)\dfrac{\partial z}{\partial y}$ is equal to

    a) 1
    b) 2
    c) 3
    d) 4

12. Derivative of $\sec^{-1}\left(\dfrac{1}{1-2x^2}\right)$ w.r.t. $\sin^{-1}(3x - 4x^3)$ is

    a) $\dfrac{1}{4}$
    b) $\dfrac{3}{2}$
    c) 1
    d) $\dfrac{2}{3}$

13. If $(x + y) \sin u = x^2 y^2$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

   a) $\sin u$ b) $\operatorname{cosec} u$ c) $2 \tan u$ d) $3 \tan u$

14. If $y = \left(1 + \frac{1}{x}\right)\left(1 + \frac{2}{x}\right)\left(1 + \frac{3}{x}\right) \cdots \left(1 + \frac{n}{x}\right)$ and $x \neq 0$, then $\frac{dy}{dx}$ when $x = -1$ is

   a) $n!$ b) $(n - 1)!$ c) $(-1)^n (n - 1)!$ d) $(-1)^n n!$

15. If $y = a \sin^3 \theta$ and $x = a \cos^3 \theta$, then at $\theta = \frac{\pi}{3}$, $\frac{dy}{dx}$ is equal to

   a) $\frac{1}{\sqrt{3}}$ b) $-\sqrt{3}$ c) $\frac{-1}{\sqrt{3}}$ d) $\sqrt{3}$

16. If y is a function of x and $\log (x + y) = 2xy$, then the value of $y'(0)$ is equal to

   a) 1 b) -1 c) 2 d) 0

17. If $f(x)$ is a polynomial of degree $n(> 2)$ and $f(x) = f(\alpha - x)$, (where α is fixed real number), then the degree of $f'(x)$ is

   a) n b) $n - 1$ c) $n - 2$ d) None of these

18. Derivative of the function $f(x) = \log_5(\log_7 x), x > 7$ is

   a) $\dfrac{1}{x(\log 5)(\log 7)(\log_7 x)}$ b) $\dfrac{1}{x(\log 5)(\log 7)}$

   c) $\dfrac{1}{x(\log x)}$ d) None of these

19. If variables x and y are related by the equation $x = \int_0^y \frac{1}{\sqrt{1+9u^2}} \, du$, then 43, $\frac{d^2 y}{dx^2}$ is equal to

   a) $\sqrt{1 + 9y^2}$ b) $\dfrac{1}{\sqrt{1 + 9y^2}}$ c) $9\,y$ d) $\frac{1}{9}y$

20. If $u = x^2 + y^2$ and $x = s + 3t, y = 2s - t$, then $\frac{d^2 u}{ds^2}$ is equal to

   a) 12 b) 32 c) 36 d) 10

21. If $f(x) = \sqrt{1 + \cos^2(x^2)}$, then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

   a) $\dfrac{\sqrt{\pi}}{6}$ b) $-\sqrt{\dfrac{\pi}{6}}$ c) $\dfrac{1}{\sqrt{6}}$ d) $\dfrac{\pi}{\sqrt{6}}$

22. If $y = 2^{\log x}$, then $\frac{dy}{dx}$ is

   a) $\dfrac{2^{\log x}}{\log 2}$ b) $2^{\log x}.\log 2$ c) $\dfrac{2^{\log x}}{x}$ d) $\dfrac{2^{\log x}.\log 2}{x}$

23. $\frac{d}{dx}\left[\log\left\{e^x \left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$ is equal to

   a) 1 b) $\dfrac{x^2 + 1}{x^2 - 4}$ c) $\dfrac{x^2 - 1}{x^2 - 4}$ d) $e^x.\dfrac{x^2 - 1}{x^2 - 4}$

24. If $f : R \to R$ is an even function which is twice differentiable on R and $f''(\pi) = 1$, then $f''(-\pi)$ is equal to

   a) -1 b) 0 c) 1 d) 2

25. If $x = \int_0^y \frac{1}{\sqrt{1+4\,t^2}} \, dt$, then $\frac{d^2 y}{dx^2}$ is

   a) $2y$ b) $4y$ c) $8y$ d) $6y$

26. Let $3f(x) - 2f\left(\frac{1}{x}\right) = x$, then $f'(2)$ is equal to

   a) $\dfrac{2}{7}$ b) $\dfrac{1}{2}$ c) 2 d) $\dfrac{7}{2}$

27. $\frac{d^n}{dx^n}(\log x)$ is equal to

   a) $\dfrac{(n - 1)!}{x^n}$ b) $\dfrac{n!}{x^n}$ c) $\dfrac{(n - 2)!}{x^n}$ d) $(-1)^{n-1}\dfrac{(n - 1)!}{x^n}$

28. Let a function $y = y(x)$ be defined parametrically by $x = 2t - |t|, y = t^2 + t|t|$. Then, $y'(x), x > 0$

   a) 0 b) $4x$ c) $2x$ d) Does not exist

29. Let $f(x) = (x^3 + 2)^{30}$. If $f^n(x)$ is a polynomial of degree 20, where $f^n(x)$ denotes the nth order derivative of $f(x)$ with respect to x, then the value of n is

    a) 60   b) 40   c) 70   d) 50

30. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \text{to } \infty}}}$, then $\frac{dy}{dx}$ is equal to

    a) $\dfrac{x}{2y - 1}$ b) $\dfrac{2}{2y - 1}$ c) $-\dfrac{1}{2y - 1}$ d) $\dfrac{1}{2y - 1}$

31. If $f(x) = x^3 + x^2 + f'(1) + xf''(2) + f'''(3)$, $\forall x \in R$ where $f(x)$ is a polynomial of degree 3, then

    a) $f(0) + f(2) = f(1)$ b) $f(0) + f(3) = 0$
    c) $f(1) + f(3) = f(2)$ d) All of these

32. If $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$, then $y'(0)$ is

    a) 1/2   b) 0   c) 1   d) Does not exist

33. If $y = e^{\sin^{-1}x}$ and $u = \log x$, then $\frac{dy}{du}$ is

    a) $\dfrac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$ b) $xe^{\sin^{-1}x}$ c) $\dfrac{xe^{\sin^{-1}x}}{\sqrt{1-x^2}}$ d) $\dfrac{e^{\sin^{-1}x}}{x}$

34. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$, then $f'\left(\dfrac{\pi}{4}\right)$ is

    a) $\sqrt{2}$ b) $\dfrac{1}{\sqrt{2}}$ c) 0   d) $\dfrac{\sqrt{3}}{2}$

35. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \infty}}}}$, then $\frac{dy}{dx}$ is equal to

    a) $\dfrac{x}{2y - 1}$ b) $\dfrac{x}{2y + 1}$ c) $\dfrac{1}{x(2y - 1)}$ d) $\dfrac{1}{x(1 - 2y)}$

36. If $y = \cos^{-1}(\cos x)$, then $\frac{dy}{dx}$ is

    a) 1 in the whole plane
    b) -1 in the whole plane
    c) 1 in the 2nd and 3rd quadrants of the plane
    d) -1 in the 3rd and 4th quadrants of the plane

37. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{dy}{dx}$ is equal to

    a) $\cos\theta$ b) $\tan\theta$ c) $\sec\theta$ d) $\csc\theta$

38. If $y = \log(\sin(x^2))$, $0 < x < \dfrac{\pi}{2}$, then $\frac{dy}{dx}$ at $x = \dfrac{\sqrt{\pi}}{2}$ is

    a) 0   b) 1   c) $\dfrac{\pi}{4}$ d) $\sqrt{\pi}$

39. The expression of $\frac{dy}{dx}$ of the function $y = a^{x^{a^{x \cdots \infty}}}$, is

    a) $\dfrac{y^2}{x(1 - y\log x)}$ b) $\dfrac{y^2\log y}{x(1 - y\log x)}$ c) $\dfrac{y^2\log y}{x(1 - y\log x\log y)}$ d) $\dfrac{y^2\log y}{x(1 + y\log x\log y)}$

40. If $f(x) = x^n + 4$, then the value of $f(1) + \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} + \ldots + \dfrac{f^n(1)}{n!}$ is

    a) 2^{n-1} b) $2^n + 4$
    c) $1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{2!} + \ldots + \dfrac{1}{n!}$ d) None of these

41. If $x = e^t\sin t$, $y = e^t\cos t$, t is a parameter, then $\frac{d^2y}{dx^2}$ at $(1, 1)$ is equal to

    a) $-\dfrac{1}{2}$ b) $-\dfrac{1}{4}$ c) 0   d) $\dfrac{1}{2}$

42. Find the derivative of y with respect to x if $e^x + e^y = e^{x+y}$

    a) $-e^{x-y}$ b) e^{x-y} c) $-e^{y-x}$ d) e^{y-x}

43. If $f'(x) = \sin(\log x)$ and $y = f\left(\dfrac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

    a) $6\sin\log(5)$ b) $5\sin\log(6)$ c) $12\sin\log(5)$ d) $5\sin\log(12)$

44. The derivative of $f(\tan x)$ w. r. t. $g(\sec x)$ at $x = \frac{\pi}{4}$, where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$, is

a) $\frac{1}{\sqrt{2}}$
b) $\sqrt{2}$
c) 1
d) None of these

45. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then y' (1) equals
   a) -1
   b) 1
   c) $\log 2$
   d) $-\log 2$

46. If $y = x \log \left(\frac{x}{a+bx}\right)$, then $\frac{x^3 d^2 y}{dx^2}$ is equal to

a) $x \frac{dy}{dx} - y$
b) $\left(x \frac{dy}{dx} - y\right)^2$
c) $y \frac{dy}{dx} - x$
d) None of these

47. The value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, where y is given by $y = x^{\sin x} + \sqrt{x}$, is

a) $1 + \frac{1}{\sqrt{2\pi}}$
b) 1
c) $\frac{1}{\sqrt{2\pi}}$
d) $1 - \frac{1}{\sqrt{2\pi}}$

48. Let $\phi(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^5}$, then $\frac{d}{dx} \phi(x)$ is equal to

a) $\frac{1}{1 + [\phi(x)]^5}$
b) $\frac{1}{1 + [f(x)]^5}$
c) $1 + [\phi(x)]^5$
d) $1 + f(x)$

49. The derivative of $\left[\frac{e^x + 1}{e^x}\right]$ is equal to

a) 0
b) $\frac{1}{e^x}$
c) $-\frac{1}{e^x}$
d) e^x

50. A value of x in the interval $(1,2)$ such that $f'(x) = 0$, where
   $f(x) = x^3 - 3x^2 + 2x + 10$ is

a) $\frac{3 + \sqrt{3}}{3}$
b) $\frac{3 + \sqrt{2}}{2}$
c) $1 + \sqrt{2}$
d) $\sqrt{2}$

51. If $x^2 + y^2 = a^2$ and $k = 1/a$, then k is equal to

a) $\frac{y''}{\sqrt{1 + y'}}$
b) $\frac{|y''|}{\sqrt{(1 + y'^2)^3}}$
c) $\frac{2y''}{\sqrt{1 + y'^2}}$
d) $\frac{y''}{2\sqrt{(1 + y'^2)^3}}$

52. The value of $\frac{d}{dx}\left[\left(\frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x \tan^2 x}\right) \cot 3x\right]$ is
   a) $\tan 2x \tan x$
   b) $\tan 3x \tan x$
   c) $\sec^2 x$
   d) $\sec x \tan x$

53. $x = \frac{1 - \sqrt{y}}{1 + \sqrt{y}} \Rightarrow \frac{dy}{dx}$ is equal to

a) $\frac{4}{(x + 1)^2}$
b) $\frac{4(x - 1)}{(1 + x)^3}$
c) $\frac{x - 1}{(1 + x)^3}$
d) $\frac{4}{(x + 1)^3}$

54. If $y = \cos^{-1}\left(\frac{2 \cos x - 3 \sin x}{\sqrt{13}}\right)$, then $\frac{dy}{dx}$ is
   a) Zero
   b) Constant $= 1$
   c) Constant $\neq 1$
   d) None of these

55. If $y = \sin(\log_e x)$, then $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx}$ is equal to
   a) $\sin(\log_e x)$
   b) $\cos(\log_e x)$
   c) y^2
   d) $-y$

56. If $y = \sin^{-1} \frac{x}{2} + \cos^{-1} \frac{x}{2}$, then the value of $\frac{dy}{dx}$ is
   a) 1
   b) -1
   c) 0
   d) 2

57. If $y = \cos(\sin x^2)$, then at $x = \sqrt{\frac{\pi}{2}}$, $\frac{dy}{dx}$ is equal to

a) -2
b) 2
c) $-2\sqrt{\frac{\pi}{2}}$
d) 0

58. If $f: R \to R$ is an even function having derivatives of all orders, then an odd function among the following is
   a) f''
   b) f'''
   c) $f' + f''$
   d) $f'' + f'''$

59. If $z = \sec^{-1}\left(\frac{x^4 + y^4 - 8x^2 y^2}{x^2 + y^2}\right)$, then $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$ is equal to

a) $\cot z$ b) $2 \cot z$ c) $2 \tan z$ d) $2 \sec z$

60. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{2^x + 2^y}{2^x - 2^y}$ b) $\dfrac{2^x + 2^y}{1 + 2^{x+y}}$ c) $2^{x-y}\left(\dfrac{2^y - 1}{1 - 2^x}\right)$ d) $\dfrac{2^{x+y} - 2^x}{2^y}$

61. If $f(x) = 10 \cos x + (13 + 2x) \sin x$, then $f''(x) + f(x) =$

a) $\cos x$ b) $4 \cos x$ c) $\sin x$ d) $4 \sin x$

62. If $xy = \tan^{-1}(xy) + \cot^{-1}(xy)$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{y}{x}$ b) $-\dfrac{y}{x}$ c) $\dfrac{x}{y}$ d) $-\dfrac{x}{y}$

63. If $x^y . y^x = 16$, then the value of $\frac{dy}{dx}$ at $(2, 2)$ is

a) -1 b) 0 c) 1 d) None of these

64. If $x = a \cos\theta$, $y = b \sin\theta$, then $\frac{d^3y}{dx^3}$ is equal to

a) $-\dfrac{3b}{a^3}\operatorname{cosec}^4\theta \cot^4\theta$ b) $\dfrac{3b}{a^3}\operatorname{cosec}^4\theta \cot^4\theta$ c) $-\dfrac{3b}{a^3}\operatorname{cosec}^4\theta \cot\theta$ d) None of these

65. If $x = \phi(t), y = \psi(t)$, then $\frac{d^2y}{dx^2}$ is equal to

a) $\dfrac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}$ b) $\dfrac{\phi'\psi'' - \psi'\phi''}{(\phi')^3}$ c) $\dfrac{\phi''}{\psi''}$ d) $\dfrac{\psi''}{\phi''}$

66. If $u = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to

a) $\sin u$ b) $\tan u$ c) $\cos u$ d) $\cot u$

67. If $y = \sec^{-1}\left(\dfrac{x+1}{x-1}\right) + \sin^{-1}\left(\dfrac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is

a) 1 b) $\dfrac{x-1}{x+1}$ c) 0 d) $\dfrac{x+1}{x-1}$

68. If $y = x^n \log x + x(\log x)^n$, then $\frac{dy}{dx}$ is equal to

a) $x^{n-1}(1 + n \log x) + (\log x)^{n-1}[n + \log x]$ b) $x^{n-2}(1 + n \log x) + (\log x)^{n-1}[n + \log x]$

c) $x^{n-1}(1 + n \log x) + (\log x)^{n-1}[n - \log x]$ d) None of the above

69. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \cdots \infty}}}}$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{y+x}{y^2-2x}$ b) $\dfrac{y^3-x}{2y^2-2xy-1}$ c) $\dfrac{y^3+x}{2y^2-x}$ d) None of these

70. If $y = \int_0^x f(t) \sin\{k(x - t)\} dt$, then $\frac{d^2y}{dx^2} + k^2 y$ equals

a) 0 b) y c) $kf(x)$ d) $k^2 f(x)$

71. If $f(x) = 10 \cos x + (13 + 2x) \sin x$, then $f''(x) + f(x)$ is equal to

a) $\cos x$ b) $4 \cos x$ c) $\sin x$ d) $4 \sin x$

72. Observe the following statements :

I. If $f(x) = ax^{41} + bx^{-40}$, then $\dfrac{f''(x)}{f(x)} = 1640\, x^{-2}$

II. $\dfrac{d}{dx}\left\{\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)\right\} = \dfrac{1}{1+x^2}$

Which of the following is correct?

a) I is true, but II is false   b) Both I and II true   c) Neither I nor II is true   d) I is false, but II is true

73. $\dfrac{d}{dx}[\cos x^0]$ is equal to

a) $-\dfrac{\pi}{180}\sin x^0$ b) $-\sin x^0$ c) $\dfrac{\pi}{180}\sin x^0$ d) $-\dfrac{\pi x}{180}\sin x$

74. If $y = e^{1+\log_e x}$, then the value of $\frac{dy}{dx}$ is equal to

a) e b) 1 c) 0 d) $\log_e x \; e^{\log_e e^x}$

75. If $y = f(x)$ and $y \cos x + x \cos y = \pi$, then the value of $f''(0)$ is

   a) π b) $-\pi$ c) 0 d) 2π

76. If $x = a \sin\theta$ and $y = b \cos\theta$, then $\frac{d^2 y}{dx^2}$ is equal to

   a) $\frac{a}{b^2} \sec^2\theta$ b) $-\frac{b}{a} \sec^2\theta$ c) $\frac{b}{a^2} \sec^3\theta$ d) $-\frac{b}{a^2} \sec^3\theta$

77. The derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\cos^{-1}\sqrt{1-x^2}$ is

   a) $\frac{\sqrt{1-x^2}}{1+x^2}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{2}{\sqrt{1-x^2}(1+x^2)}$ d) $\frac{2\sqrt{1-x^2}}{1+x^2}$

78. If $y = \sin^{-1}\left\{\sqrt{x-ax} - \sqrt{a-ax}\right\}$, then $\frac{dy}{dx}$ is equal to

   a) $\frac{1}{\sin\sqrt{a-ax}}$ b) $\sin\sqrt{x}\sin\sqrt{a}$ c) $\frac{1}{2}\sqrt{x(1-x)}$ d) 0

79. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

   a) -1 b) 1 c) 2 d) 4

80. Let $f(x) = e^x$, $g(x) = \sin^{-1} x$ and $h(x) = f[g(x)]$, then $\frac{h'(x)}{h(x)}$ is equal to

   a) $e^{\sin^{-1} x}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\sin^{-1} x$ d) $\frac{1}{(1-x^2)}$

81. If $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$, then $\frac{dy}{dx}$ is equal to

   a) $\cot\frac{\theta}{2}$ b) $\tan\frac{\theta}{2}$ c) $\frac{1}{2}\operatorname{cosec}^2\frac{\theta}{2}$ d) $-\frac{1}{2}\operatorname{cosec}^2\frac{\theta}{2}$

82. A curve is given by the equations $x = a\cos\theta + \frac{1}{2}b\cos 2\theta$, $y = a\sin\theta + \frac{1}{2}b\sin 2\theta$. Then the points for which $\frac{d^2 y}{dx^2} = 0$ are given by

   a) $\sin\theta = \frac{2a^2+b^2}{5ab}$ b) $\tan\theta = \frac{3a^2+2b^2}{4ab}$ c) $\cos\theta = -\frac{a^2+2b^2}{3ab}$ d) None of these

83. If $y = \log_a x + \log_x a + \log_x x + \log_a a$, then $\frac{dy}{dx}$ is equal to

   a) $\frac{1}{x} + x\log a$ b) $\frac{\log a}{x} + \frac{x}{\log a}$ c) $\frac{1}{x\log a} + x\log a$ d) $\frac{1}{x\log a} - \frac{\log a}{x(\log x)^2}$

84. If $y = \sin[\cos^{-1}\{\sin(\cos^{-1} x)\}]$, then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is equal to

   a) 0 b) -1 c) $\frac{2}{\sqrt{3}}$ d) 1

85. If $8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$ and $y = x^2 f(x)$, then $\frac{dy}{dx}$ at $x = -1$ is equal to

   a) 0 b) $\frac{1}{14}$ c) $-\frac{1}{14}$ d) 1

86. If $x = \sin t$, $y = \cos pt$, then

   a) $(1-x^2)y_2 + xy_1 + p^2 y = 0$ b) $(1-x^2)y_2 + xy_1 - p^2 y = 0$

   c) $(1+x^2)y_2 - xy_1 + p^2 y = 0$ d) $(1-x^2)y_2 - xy_1 + p^2 y = 0$

87. The derivative of $y = (1-x)(2-x)\ldots(n-x)$ at $x = 1$ is equal to

   a) 0 b) $(-1)(n-1)!$ c) $n!-1$ d) $(-1)^{n-1}(n-1)!$

88. If $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$, then $f'(a)$ is equal to

   a) -1 b) 1 c) 0 d) a

89. If $y = \log_2 \log_2(x)$, then $\frac{dy}{dx}$ is equal to

   a) $\frac{\log_2 e}{\log_e x}$ b) $\frac{\log_2 e}{x\log_x 2}$ c) $\frac{\log_2 x}{\log_e 2}$ d) $\frac{\log_2 e}{x\log_e x}$

90. The differential coefficient of $f(\sin x)$ with respect to x, where $f(x) = \log x$, is

a) $\tan x$ b) $\cot x$ c) $f(\cos x)$ d) $\frac{1}{x}$

91. If $y = e^{ax}\sin bx$, then $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + a^2y$ is equal to

a) 0   b) 1   c) $-b^2y$ d) $-by$

92. If $f(x) = (\log_{\cot x}\tan x)(\log_{\tan x}\cot x)^{-1} + \tan^{-1}\left(\frac{4x}{\sqrt{4-x^2}}\right)$, then $f'(0)$ is equal to

a) 2   b) 0   c) 1/2   d) -2

93. $f(x)$ is a polynomial of degree 2, $f(0) = 4, f'(0) = 3$ and $f''(0) = 4$, then $f(-1)$ is equal to

a) 3   b) -2 c) 2   d) -3

94. If $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$, then $\frac{dy}{dx}$ equal

a) $2x\{1 + \tan(\log_e x)\} + x\sec^2(\log_e x)$

b) $x\{1 + \tan(\log_e x)\} + \sec^2(\log_e x)$

c) $2x\{1 + \tan(\log_e x)\} + x^2\sec^2(\log_e x)$

d) $2x\{1 + \tan(\log_e x)\} + \sec^2(\log_e x)$

95. If $y^2 = ax^2 + bx + c$ where a, b, c are constants, then $y^3\frac{d^2y}{dx^2}$, is equal to

a) A constant

b) A function of x

c) A function of y

d) A function of x and y both

96. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is

a) 0   b) 1   c) $\frac{1}{1-x^2}$ d) $\frac{1}{1+x^2}$

97. If $I_n = \frac{d^n}{dx^n}(x^n\log x)$, then $I_n - nI_{n-1}$ is equal to

a) n b) $n-1$ c) $n!$ d) $(n-1)!$

98. If $y = \sin(\log_e x)$, then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to

a) $\sin(\log_e x)$ b) $\cos(\log_e x)$ c) y^2 d) $-y$

99. Let $y = t^{10} + 1$ and $x = t^8 + 1$, then $\frac{d^2y}{dx^2}$ is equal to

a) $\frac{5}{2}t$ b) $20\,t^8$ c) $\frac{5}{16t^6}$ d) None of these

100. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ with respect to $\sin^{-1}(3x - 4x^3)$ is

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{1}{2}$ d) 1

101. If $y = (1 + x)(1 + x^2)(1 + x^4)\ldots(1 + x^{2^n})$, then the value of $\frac{dy}{dx}$ at $x = 0$ is

a) 0   b) -1 c) 1   d) None of these

102. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then $\frac{dy}{dx}$ equals

a) $\sqrt{(1-x^2)(1-y^2)}$ b) $\sqrt{\frac{1-y^2}{1-x^2}}$ c) $\sqrt{\frac{1-x^2}{1-y^2}}$ d) None of these

103. The derivative of e^{x^3} with respect to $\log x$ is

a) e^{x^3} b) $3x^2 2e^{x^3}$ c) $3x^3 e^{x^3}$ d) $3x^2 e^{x^3} + 3x^2$

104. The rate of change of $\sqrt{(x^2 + 16)}$ with respect to $\frac{x}{x-1}$ at $x = 3$ is

a) 2   b) $\frac{11}{5}$ c) $-\frac{12}{5}$ d) -3

105. If $x = \log(1 + t^2)$ and $y = t - \tan^{-1} t$. Then, $\frac{dy}{dx}$ is equal to

a) $e^x - 1$ b) $t^2 - 1$ c) $\frac{\sqrt{e^x - 1}}{2}$ d) $e^x - y$

106. If $y = (1 + x)(1 + x^2)(1 + x^4) \dots (1 + x^{2n})$, then the value of $\left(\frac{dy}{dx}\right)_{x=0}$ is

a) 0   b) -1 c) 1   d) 2

107. If $f(x) = (1 - x)^n$, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \dots \frac{f^n(0)}{n!}$, is

a) 2^n b) 0   c) 2^{n-1} d) None of these

108. If $f(x) = (\log_{\cot x} \tan x)(\log_{\tan x} \cot x)^{-1} + \tan^{-1} \frac{4x}{4 - x^2}$, then $f'(2)$ is equal to

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 1   d) -1

109. If $y = \cos^2 \frac{3x}{2} - \sin^2 \frac{3x}{2}$, then $\frac{d^2 y}{dx^2}$ is

a) $-3\sqrt{1 - y^2}$ b) $9y$ c) $-9y$ d) $3\sqrt{1 - y^2}$

110. If $f(x) = \log_x(\log_e x)$, then $f'(x)$ at $x = e$ is equal to

a) 1   b) 2   c) 0   d) $\frac{1}{e}$

111. If $f(x) = \cos^{-1}\left\{\frac{1 + (\log_e x)^2}{1 + (\log_e x)^2}\right\}$, then $f'(e)$

a) Does not exist   b) Is equal to $\frac{2}{e}$ c) Is equal to $\frac{1}{e}$ d) Is equal to 1

112. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \dots + x^n$, then $f''(1)$ is equal to

a) $n(n-1)2^{n-1}$ b) $(n-1)2^{n-1}$ c) $n(n-1)2^{n-2}$ d) $n(n-1)2^n$

113. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{(2^x + 2^y)}{(2^x - 2^y)}$ b) $\frac{(2^x + 2^y)}{(1 + 2^{x+y})}$ c) $2^{x-y}\left(\frac{2^y - 1}{1 - 2^x}\right)$ d) $\frac{2^{x+y} - 2^x}{2^y}$

114. If $2x^2 - 3xy + y^2 + x + 2y - x = 0$, then $\frac{dy}{dx} =$

a) $\frac{3y - 4x - 1}{2y - 3x + 2}$ b) $\frac{3y + 4x + 1}{2y + 3x + 2}$ c) $\frac{3y - 4x + 1}{2y - 3x - 2}$ d) $\frac{3y - 4x + 1}{2y + 3x + 2}$

115. If $\sin y + e^{-x \cos y} = e$, then $\frac{dy}{dx}$ at $(1, \pi)$ is

a) $\sin y$ b) $-x \cos y$ c) e d) $\sin y - x \cos y$

116. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a - x}{1 + ax}\right)\right]$ is equal to

a) $-\frac{1}{1 + x^2}$ b) $\frac{1}{1 + a^2} - \frac{1}{1 + x^2}$ c) $\frac{1}{1 + \left(\frac{a - x}{1 + ax}\right)^2}$ d) $\frac{-1}{\sqrt{1 - \left(\frac{a - x}{1 + ax}\right)^2}}$

117. If $y = \tan^{-1}\left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right)$, then $\frac{dy}{dx}$ is equal to

a) 2   b) -1 c) $\frac{a}{b}$ d) $\frac{b}{a}$

118. Let $y = x^{x^{x^{\dots}}}$, then $\frac{dy}{dx}$ is equal to

a) yx^{y-1} b) $\frac{y^2}{x(1 - y \log x)}$ c) $\frac{y}{x(1 + y \log x)}$ d) None of these

119. If $x^x y^y z^z = c$, then $\frac{\partial z}{\partial x}$ is equal to

a) $\left(\frac{1 + \log x}{1 + \log z}\right)$ b) $-\left(\frac{1 + \log x}{1 + \log z}\right)$ c) $\left(\frac{1 + \log z}{1 + \log x}\right)$ d) None of these

120. If $y = \sec^{-1}\left(\frac{\sqrt{x} + 1}{\sqrt{x} - 1}\right) + \sin^{-1}\left(\frac{\sqrt{x} - 1}{\sqrt{x} + 1}\right)$, then $\frac{dy}{dx}$ equals

a) 1   b) 0   c) $\frac{\sqrt{x} + 1}{\sqrt{x} - 1}$ d) $\frac{\sqrt{x} - 1}{\sqrt{x} + 1}$

121. If $f(x) = \sqrt{x^2 - 2x + 1}$, then

a) $f'(x) = 1$ for all x

b) $f'(x) = -1$ for all $x \leq 1$

c) $f'(x) = 1$ for all $x \geq 1$

d) None of these

122. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

a) 0   b) 1   c) 2   d) None of these

123. If $u(x,y) = y\log x + x\log y$, then $u_x u_y - u_x\log x - u_y\log y + \log x\log y$ is equal to

a) 0   b) -1 c) 1   d) 2

124. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

a) $-16x$ b) $16x$ c) x d) $-x$

125. If $f(x)$ has a derivative at $x = a$, then $\lim_{x \to a}\frac{xf(a)-af(x)}{x-a}$ is equal to

a) $f(a) - af'(a)$ b) $af(a) - f'(a)$ c) $f(a) + f'(a)$ d) $af(a) + f'(a)$

126. $\frac{d}{dx}[x^x + x^a + a^x + a^a] = \cdots$, a is constant

a) $x^x(1 + \log x) + a.x^{a-1}$

b) $x^x(1 + \log x) + a.x^{a-1} + a^x\log a$

c) $x^x(1 + \log x) + a^a \ (1 + \log a)$

d) $x^x(1 + \log x) + a^a(1 + \log a) + ax^{a-1} + a^a(1 + \log a)$

127. If $y = a^x.b^{2x-1}$, then $\frac{d^2y}{dx^2}$ is

a) $y^2\log ab^2$ b) $y\log ab^2$ c) y^2 d) $y(\log ab^2)^2$

128. If $f(x) = x + 2$, then the value of $f'[f(x)]$ at $x = 4$ is

a) 8   b) 1   c) 4   d) 5

129. If $ax^2 + 2hxy + by^2 = 1$, then $\frac{d^2y}{dx^2}$ equals

a) $\frac{h^2 + ab}{(hx + by)^3}$ b) $\frac{h^2 - ab}{(hx + by)^2}$ c) $\frac{h^2 + ab}{(hx + by)^3}$ d) $\frac{h^2 - ab}{(hx + by)^3}$

130. Let $f(x) = \sin x, g(x) = x^2$ and $h(x) = \log_e x$. If $F(x) = (hogof)(x)$, then $F''(x)$ is equal to

a) $a\,\text{cosec}^3 x$ b) $2\cot x^2 - 4x^2\,\text{cosec}^2 x^2$

c) $2x\cot x^2$ d) $-2\,\text{cosec}^2 x$

131. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{x}{y}$ b) $-\frac{x}{y}$ c) $\frac{y}{x}$ d) $-\frac{y}{x}$

132. If $y = \tan^{-1}(\sec x - \tan x)$, then $\frac{dy}{dx}$ is

a) 2   b) -2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

133. If $x = e^t\sin t, y = e^t\cos t$, then $\frac{d^2y}{dx^2}$ at $x = \pi$, is

a) $2e^\pi$ b) $\frac{1}{2}e^\pi$ c) $\frac{1}{2e^\pi}$ d) $\frac{2}{e^\pi}$

134. The derivative of $f(x) = 3|2 + x|$ at the point $x_0 = -3$, is

a) 3   b) -3 c) 0   d) Does not exist

135. If variables x and y are related by the equation $x = \int_0^y \frac{1}{\sqrt{1+9u^2}}\,du$, then $\frac{d^2y}{dx^2}$ is equal to

a) $\sqrt{1 + 9y^2}$ b) $\frac{1}{1 + 9y^2}$ c) $9y$ d) $\frac{1}{9}y$

136. If $f(x) = x\tan^{-1} x$, then $f'(1)$ is equal to

a) $\frac{1}{2} + \frac{\pi}{4}$ b) $-\frac{1}{2} + \frac{\pi}{4}$ c) $-\frac{1}{2} - \frac{\pi}{4}$ d) $\frac{1}{2} - \frac{\pi}{4}$

137. If $f: (-1,1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then, $g'(0)$ is equal to

a) 4   b) -4 c) 0   d) -2

138. The differential coefficient of the function $|x - 1| + |x - 3|$ at the point $x = 2$ is

a) -2 b) 0 c) 2 d) undefined

139. If $y = \tan^{-1}(\sec x - \tan x)$, then $\frac{dy}{dx}$ is equal to

a) 2 b) -2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

140. If $x = a\cos^4\theta, y = a\sin^4\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{3\pi}{4}$ is

a) -1 b) 1 c) $-a^2$ d) a^2

141. If $\sec\left(\frac{x^2-y^2}{x^2+y^2}\right) = e^a$, then $\frac{dy}{dx}$ is equal to

a) $\frac{y^2}{x^2}$ b) $\frac{y}{x}$ c) $\frac{x}{y}$ d) $\frac{x^2-y^2}{x^2+y^2}$

142. If $y = \frac{a+bx^{3/2}}{x^{5/4}}$ and $y' = 0$ at $x = 5$, then the ratio $a:b$ is equal to

a) $\sqrt{5}:1$ b) $5:2$ c) $3:5$ d) $1:2$

143. If $f(x) = be^{ax} + ae^{bx}$, then $f''(0)$ is equal to

a) 0 b) $2ab$ c) $ab(a+b)$ d) ab

144. If $x^m y^n = (x+y)^{m+n}$, then $(dy/dx)_{x=1,y=2}$ is equal to

a) $1/2$ b) 2 c) $2m/n$ d) $m/2n$

145. If $f(x)$ and $g(x)$ are two functions with $g(x) = x - \frac{1}{x}$ and

$fog(x) = x^3 - \frac{1}{x^3}$, then $f'(x)$ is

a) $3x^2 + 3$ b) $x^2 - \frac{1}{x^2}$ c) $1 + \frac{1}{x^2}$ d) $3x^2 + \frac{3}{x^4}$

146. If a curve is given by $x = a\cos t + \frac{b}{2}\cos 2t$ and $y = a\sin t + \frac{b}{2}\sin 2t$, then the points for which $\frac{d^2y}{dx^2} = 0$ are given by

a) $\sin t = \frac{2a^2+b^2}{3ab}$ b) $\cos t = -\frac{a^2+b^2}{3ab}$ c) $\tan t = a/b$ d) None of these

147. Let f be a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If

$h'(x) = [f(x)^2 + g(x)^2]h(1) = 8$ and $h(0) = 2$, then $h(2)$ is equal to

a) 1 b) 2 c) 3 d) None of these

148. If $y = \log x^x$, then the value of $\frac{dy}{dx}$ is

a) $x^x(1 + \log x)$ b) $\log(ex)$ c) $\log\left(\frac{e}{x}\right)$ d) $\log\left(\frac{x}{e}\right)$

149. If $f''(x) = -f(x)$, where $f(x)$ is a continuous double differentiable function and $g(x) = f'(x)$. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and $F(5) = 5$ then $F(10)$ is

a) 0 b) 5 c) 10 d) 25

150. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{\cos x}{2y-1}$ b) $\frac{-\cos x}{2y-1}$ c) $\frac{\sin x}{1-2y}$ d) $\frac{-\sin x}{1-2y}$

151. If $y = x^2 e^{mx}$, where m is a constant, then $\frac{d^3y}{dx^3}$ is equal to

a) $me^{mx}(m^2x^2 + 6mx + 6)$ b) $2m^3 x e^{mx}$

c) $me^{mx}(m^2x^2 + 2mx + 2)$ d) None of these

152. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{x^2y^3}$ b) $\frac{1}{xy^3}$ c) $\frac{1}{x^2y^2}$ d) $\frac{1}{x^3y}$

153. If $x = \sin^{-1}(3t - 4t^3)$ and $y = \cos^{-1}(\sqrt{1-t^2})$, then $\frac{dy}{dx}$ is equal to

a) $1/2$ b) $2/5$ c) $3/2$ d) $1/3$

154. If $f(x) = \sqrt{1 - \sin 2x}$, then $f'(x)$ equals
a) $-(\cos x + \sin x)$, for $x \in (\pi/4, \pi/2)$
b) $\cos x + \sin x$, for $x \in (0, \pi/4)$
c) $-(\cos x + \sin x)$, for $x \in (0, \pi/4)$
d) $\cos x - \sin x$, for $x \in (\pi/4, \pi/2)$

155. Derivative of $\sin x$ w.r.t. $\cos x$ is
a) $\cos x$ b) $\cot x$ c) $-\cot x$ d) $\tan x$

156. The derivative of $F[f\{\phi(x)\}]$ is
a) $F'[f\{\phi(x)\}]$
b) $F'[f\{\phi(x)\}]f\{\phi(x)\}$
c) $F'[f\{\phi(x)\}]f'\{\phi(x)\}$
d) $F'[f\{\phi(x)\}]f'\{\phi(x)\}\phi'(x)$

157. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx}$ is equal to
a) $\frac{1}{(1+x)^2}$ b) $-\frac{1}{(1+x)^2}$ c) $\frac{1}{1+x^2}$ d) $\frac{1}{1-x^2}$

158. If $\sec\left(\frac{x^2-y^2}{x^2+y^2}\right) = e^a$, then $\frac{dy}{dx}$ is equal to
a) $\frac{y^2}{x^2}$ b) $\frac{y}{x}$ c) $\frac{x}{y}$ d) $\frac{x^2-y^2}{x^2+y^2}$

159. If $f(x) = \log_a(\log_a x)$, then $f'(x)$ is
a) $\frac{\log_a e}{x \log_e x}$ b) $\frac{\log_e a}{x \log_a x}$ c) $\frac{\log_e a}{x}$ d) $\frac{x}{\log_e a}$

160. If $y = \log^n x$, where $\log^n$ means $\log \log \log \ldots$(repeated n times), then
$x \log x \log^2 x \log^3 x \ldots \log^{n-1} x \log^n x \frac{dy}{dx}$ is equal to
a) $\log x$ b) $\log^n x$ c) $\frac{1}{\log x}$ d) 1

161. If $y = \sin px$ and y_n is the nth derivative of y, then $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ is equal to
a) 1   b) 0   c) -1 d) None of these

162. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ldots$, then $\frac{d^2y}{dx^2}$ is equal to
a) x b) $-x$ c) $-y$ d) y

163. If $f(4) = 4, f'(4) = 1$, then $\lim\limits_{x \to 4} \frac{2 - \sqrt{f(x)}}{2 - \sqrt{x}}$ is equal to
a) -1 b) 1   c) 2   d) -2

164. If $x = \frac{2at}{1+t^3}$ and $y = \frac{2at^2}{(1+t^3)^2}$, then $\frac{dy}{dx}$ is
a) ax b) a^2x^2 c) $\frac{x}{a}$ d) $\frac{x}{2a}$

165. $y = e^{a \sin^{-1} x} \Rightarrow (1 - x^2) y_{n+2} - (2n + 1)xy_{n+1}$ is equal to
a) $-(n^2 + a^2) y_n$ b) $(n^2 - a^2)y_n$ c) $(n^2 + a^2)y_n$ d) $-(n^2 - a^2)y_n$

166. If $\sec\left(\frac{x-y}{x+y}\right) = a$, then $\frac{dy}{dx}$ is
a) $\frac{y}{x}$ b) $-\frac{y}{x}$ c) $\frac{x}{y}$ d) $-\frac{x}{y}$

167. If $y = x + x^2 + x^3 \ldots \infty$, where $|x| < 1$, then for $|y| < 1, \frac{dx}{dy}$ is equal to
a) $y + y^2 + y^3 + \ldots \infty$
b) $1 - y + y^2 - y^3 + \ldots \infty$
c) $1 - 2y + 3y^2 - \ldots \infty$
d) $1 + 2y + 3y^2 + \ldots \infty$

168. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$ is equal to
a) $\sec^2 x$ b) $-\sec^2\left(\frac{\pi}{4} - x\right)$ c) $\sec^2\left(\frac{\pi}{4} + x\right)$ d) $\sec^2\left(\frac{\pi}{4} - x\right)$

169. Derivative of $\log_{10} x$ with respect to x^2 is

a) $2x^2 \log_e 10$ b) $\dfrac{\log_{10} e}{2x^2}$ c) $\dfrac{\log_e 10}{2x^2}$ d) $x^2 \log_e 10$

170. If $y = \ln\left(\dfrac{x}{a+bx}\right)^x$, then $x^3 \dfrac{d^2 y}{dx^2}$ is equal to

a) $\left(\dfrac{dy}{dx} + x\right)^2$

b) $\left(\dfrac{dy}{dx} - y\right)^2$

c) $\left(x\dfrac{dy}{dx} + y\right)^2$

d) $\left(x\dfrac{dy}{dx} - y\right)^2$

171. If f be a polynomial, then the second derivative of $f(e^x)$ is
a) $f'(e^x)$
b) $f''(e^x)e^x + f'(e^x)$
c) $f''(e^x)e^{2x} + f''(e^x)$
d) $f''(e^x)e^{2x} + f'(e^x)e^x$

172. If $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{3y - 4x - 1}{2y - 3x + 2}$ b) $\dfrac{3y + 4x + 1}{2y + 3x + 2}$ c) $\dfrac{3y - 4x + 1}{2y - 3x - 2}$ d) $\dfrac{3y - 4x + 1}{2y + 3x + 2}$

173.
If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots \infty}}}$, then $(2y - 1)\dfrac{dy}{dx}$ is equal to

a) $\sin x$ b) $-\cos x$ c) $\cos x$ d) $-\sin x$

174. If $2^x + 2^y = 2^{x+y}$, then the value of $\dfrac{dy}{dx}$ at $x = y = 1$, is

a) 0 b) -1 c) 1 d) 2

175. If $y = \sec(\tan^{-1} x)$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{x}{\sqrt{1+x^2}}$ b) $-\dfrac{x}{\sqrt{1+x^2}}$ c) $\dfrac{x}{\sqrt{1-x^2}}$ d) None of these

176. Let $f(x) = (x-7)^2(x-2)^7$, $x \in [2,7]$. The value of $\theta \in (2,7)$ such that $f'(\theta) = 0$ is equal to

a) $\dfrac{49}{4}$ b) $\dfrac{53}{9}$ c) $\dfrac{53}{7}$ d) $\dfrac{49}{9}$

177. If $x^2 + y^2 = t - \dfrac{1}{t}$ and $x^4 + y^4 = t^2 + \dfrac{1}{t^2}$, then $x^3 y \dfrac{dy}{dx}$ equals

a) 0 b) 1 c) -1 d) None of these

178. The value of $\dfrac{d}{dx}(|x-1| + |x-5|)$ at $x = 3$, is

a) -2 b) 0 c) 2 d) 4

179. If $\sin^{-1} x + \sin^{-1} y = \dfrac{\pi}{2}$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{x}{y}$ b) $-\dfrac{x}{y}$ c) $\dfrac{y}{x}$ d) $-\dfrac{y}{x}$

180. If $f(x) = \dfrac{1}{1-x}$, then the derivative of the composite function $f[f\{f(x)\}]$ is equal to

a) 0 b) $\dfrac{1}{2}$ c) 1 d) 2

181. If $x = \exp\left\{\tan^{-1}\left(\dfrac{y-x^2}{x^2}\right)\right\}$, then $\dfrac{dy}{dx}$ equals

a) $2x[1 + \tan(\log x)] + x\sec^2(\log x)$ b) $x[1 + \tan(\log x)] + \sec^2(\log x)$
c) $2x[1 + \tan(\log x)] + x^2\sec^2(\log x)$ d) $2x[1 + \tan(\log x)] + \sec^2(\log x)$

182. If $\sin y = x\sin(a+y)$, then $\dfrac{dy}{dx}$ is

a) $\dfrac{\sin a}{\sin^2(a+y)}$ b) $\dfrac{\sin^2(a+y)}{\sin a}$ c) $\sin a \sin^2(a+y)$ d) $\dfrac{\sin^2(a-y)}{\sin a}$

183. $f(x) = e^x \sin x$, then $f''(x)$ is equal to
a) $e^{6x} \sin 6x$ b) $2e^x \cos x$ c) $8e^x \sin x$ d) $8e^x \cos x$

184. If $f(x) = \cos x . \cos 2x . \cos 4x . \cos 8x . \cos 16x$, then the value of $f'\left(\dfrac{\pi}{4}\right)$ is

a) 1 b) $\sqrt{2}$ c) $\dfrac{1}{\sqrt{2}}$ d) 0

185. If $\sec^{-1}\left(\dfrac{1+x}{1-x}\right) = a$, then $\dfrac{dy}{dx}$ is

a) $\dfrac{y-1}{x+1}$
b) $\dfrac{y+1}{x-1}$
c) $\dfrac{x-1}{y-1}$
d) $\dfrac{x-1}{y+1}$

186. The derivative of $a^{\sec x}$ w. r. t. $a^{\tan x}$ $(a > 0)$ is

a) $\sec x \, a^{\sec x - \tan x}$
b) $\sin x \, a^{\tan x - \sec x}$
c) $\sin x \, a^{\sec x - \tan x}$
d) $a^{\sec x - \tan x}$

187. If $x = a\left\{\cos\theta + \log\tan\left(\dfrac{\theta}{2}\right)\right\}$ and $y = a\sin\theta$, then $\dfrac{dy}{dx}$ is equal to

a) $\cot\theta$
b) $\tan\theta$
c) $\sin\theta$
d) $\cos\theta$

188. If $\phi(x)$ is the inverse of the function $f(x)$ and $f'(x) = \dfrac{1}{1+x^5}$, then $\dfrac{d}{dx}\phi(x)$ is

a) $\dfrac{1}{1+\{\phi(x)\}^5}$
b) $\dfrac{1}{1+\{f(x)\}^5}$
c) $1 + \{\phi(x)\}^5$
d) $1 + f(x)$

189. If $f(x) = 3e^{x^2}$, then $f'(x) - 2x\,f(x) + \dfrac{1}{3}f(0) - f'(0)$ is equal to

a) 0
b) 1
c) $(7/3)e^{x^2}$
d) e^{x^2}

190. If $u = \log\left(\dfrac{x^2+y^2}{x+y}\right)$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is

a) -1
b) 0
c) 1
d) 2

191. If $F(x) = \dfrac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt$, then $F'(4)$ equals

a) $\dfrac{32}{9}$
b) $\dfrac{64}{3}$
c) $\dfrac{64}{9}$
d) $\dfrac{32}{3}$

192. If $y = \tan^{-1}\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{x^2}{\sqrt{1-x^4}}$
b) $\dfrac{x^2}{\sqrt{1+x^4}}$
c) $\dfrac{x}{\sqrt{1+x^4}}$
d) $\dfrac{x}{\sqrt{1-x^4}}$

193. If $f(x) = |x^2 - 5x + 6|$, then $f'(x)$ equals

a) $2x - 5$ for $2 < x < 3$
b) $5 - 2x$ for $2 < x < 3$
c) $2x - 5$ for $2 \le x \le 3$
d) $5 - 2x$ for $2 \le x \le 3$

194. If $\sqrt{x+y} + \sqrt{y-x} = c$, then $\dfrac{d^2y}{dx^2}$ equals

a) $2/c$
b) $-2/c^2$
c) $2/c^2$
d) $-2/c$

195. Derivative of $x^6 + 6^x$ with respect to x is

a) $12x$
b) $x + 4$
c) $6x^5 + 6^x \log 6$
d) $6x^5 + x6^{x-1}$

196. If $f(x) = \cos^2 x + \cos^2\left(x + \dfrac{\pi}{3}\right) + \sin x \sin\left(x + \dfrac{\pi}{3}\right)$ and $g\left(\dfrac{5}{4}\right) = 3$, then $\dfrac{d}{dx}\big(gof(x)\big) =$

a) 1
b) 0
c) -1
d) None of these

197. If $y = \sin^{-1}\dfrac{x}{2} + \cos^{-1}\dfrac{x}{2}$, then the value of $\dfrac{dy}{dx}$ is

a) 1
b) -1
c) 0
d) 2

198. If $z = y + f(v)$, where $v = \left(\dfrac{x}{y}\right)$, then $v\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y}$ is

a) -1
b) 1
c) 0
d) 2

199. If $y = \tan^{-1}\left(\dfrac{\sqrt{x}-x}{1+x^{3/2}}\right)$, then $y'(1)$ is equal to

a) 0
b) $\dfrac{1}{2}$
c) -1
d) $-\dfrac{1}{4}$

200. If $f(1) = 1$ and $f'(1) = 2$, then $\displaystyle\lim_{x\to1}\dfrac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ equals

a) 2
b) 4
c) 1
d) $1/2$

201. The derivative of $\cos^3 x$ w.r.t. $\sin^3 x$ is

a) $-\cot x$
b) $\cot x$
c) $\tan x$
d) $-\tan x$

202. If $y = \log\left\{\left(\dfrac{1+x}{1-x}\right)^{1/4}\right\} - \dfrac{1}{2}\tan^{-1} x$, then $\dfrac{dy}{dx} =$

a) $\dfrac{x}{1-x^2}$
b) $\dfrac{x^2}{1-x^4}$
c) $\dfrac{x}{1+x^4}$
d) $\dfrac{x}{1-x^4}$

203. If $y = \left(x + \sqrt{1+x^2}\right)^n$, then $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is

a) $n^2 y$
b) $-n^2 y$
c) $-y$
d) $2x^2 y$

204. The value of $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]$ is

a) $\frac{1}{2(1+x)\sqrt{x}}$ b) $\frac{3}{(1+x)\sqrt{x}}$ c) $\frac{2}{(1+x)\sqrt{x}}$ d) $\frac{3}{2(1+x)\sqrt{x}}$

205. If $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{2}$ b) $\frac{\pi}{4}$ c) 0 d) 1

206. $x = \cos\theta, y = \sin 5\theta \Rightarrow (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$ is

a) $-5y$ b) $5y$ c) $25y$ d) $-25y$

207. If the function $f(x)$ is defined by $f(x) = a + bx$ and $f^r = fff\ldots$(repeated r times), then $f^r(x)$ is equal to

a) $a + b^r x$ b) $ar + b^r x$ c) $ar + bx^r$ d) $a\left(\frac{b^r - 1}{b - 1}\right) + b^r x$

208. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to

a) $(1 + \log x)^{-1}$ b) $(1 + \log x)^{-2}$ c) $\log x \cdot (1 + \log x)^{-2}$ d) None of these

209. The derivative of $\sin^2 x$ with respect to $\cos^2 x$ is

a) $\tan^2 x$ b) $\tan x$ c) $-\tan x$ d) None of these

210. If $x^p y^q = (x + y)^{p+q}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{y}{x}$ b) $\frac{py}{qx}$ c) $\frac{x}{y}$ d) $\frac{qy}{px}$

211. If $y = (1 + x^2)\tan^{-1} x - x$, then $\frac{dy}{dx}$ is equal to

a) $\tan^{-1} x$ b) $2x\tan^{-1} x$ c) $2x\tan^{-1} x - 1$ d) $\frac{2x}{\tan^{-1} x}$

212. The derivative of $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ with respect to x is

a) $-\frac{1}{2\sqrt{1-x^2}}$ b) $\frac{1}{2\sqrt{1-x^2}}$ c) $\frac{2}{\sqrt{1-x^2}}$ d) $\frac{-2}{\sqrt{1-x^2}}$

213. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ is

a) $\sqrt{1-x^2}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{2\sqrt{1-x^2}}$ d) x

214. If $y = \tan^{-1} x + \cot^{-1} x + \sec^{-1} + \text{cosec}^{-1} x$, then $\frac{dy}{dx}$ is equal to

a) $\frac{x^2 - 1}{x^2 + 1}$ b) π c) 0 d) 1

215. If $y = \left(\frac{ax+b}{cx+d}\right)$, then $2\frac{dy}{dx}\cdot\frac{d^3y}{dx^3}$ is equal to

a) $\left(\frac{d^2y}{dx^2}\right)^2$ b) $3\frac{d^2y}{dx^2}$ c) $3\left(\frac{d^2y}{dx^2}\right)^2$ d) $3\frac{d^2x}{dy^2}$

216. If $y = (\log_{\cos x}\sin x)(\log_{\sin x}\cos x) + \sin^{-1}\frac{2x}{1+x^2}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is equal to

a) $\frac{8}{(4+\pi^2)}$ b) 0 c) $-\frac{8}{(4+\pi^2)}$ d) None of the above

217. If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, then $\frac{dy}{dx}$ is equal to

a) 2 b) -1 c) $\frac{a}{b}$ d) 0

218. If $x = \cos\theta, y = \sin 5\theta$, then $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} =$

a) $-5y$ b) $5y$ c) $25y$ d) $-25y$

219. The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ is

a) $\sqrt{1-x^2}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{2\sqrt{1-x^2}}$ d) x

220. If $f(x) = (x-2)(x-4)(x-6)\ldots(x-2n)$, then $f'(2)$ is
   a) $(-1)^n 2^{n-1}(n-1)!$
   b) $(-2)^{n-1}(n-1)!$
   c) $(-2)^n n!$
   d) $(-1)^{n-1} 2^n (n-1)!$

221. Find $\frac{dy}{dx}$, if $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$.
   a) $\tan\dfrac{3\theta}{2}$
   b) $-\tan\dfrac{3\theta}{2}$
   c) $\cot\dfrac{3\theta}{2}$
   d) $-\cot\dfrac{3\theta}{2}$

222. Let $f(x) = 2^{2x-1}$ and $\phi(x) - 2^x + 2x \log 2$. If $f'(x) > \phi'(x)$, then
   a) $0 < x < 1$
   b) $0 \le x < 1$
   c) $x > 0$
   d) $x \ge 0$

223. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx} =$
   a) $\dfrac{1}{(1+x)^2}$
   b) $-\dfrac{1}{(1+x)^2}$
   c) $\dfrac{1}{1+x^2}$
   d) $\dfrac{1}{1-x^2}$

224. If $y = e^{(1/2)\log(1+\tan^2 x)}$, then $\frac{dy}{dx}$ is equal to
   a) $\dfrac{1}{2}\sec^2 x$
   b) $\sec^2 x$
   c) $\sec x \tan x$
   d) $e^{1/2\,\log(1+\tan^2 x)}$

225. If $f(x) = \frac{x-1}{4} + \frac{(x-1)^3}{12} + \frac{(x-1)^5}{20} + \frac{(x-1)^7}{28} + \cdots$, where $0 < x < 2$, then $f'(x)$ is equal to
   a) $\dfrac{1}{4x(2-x)}$
   b) $\dfrac{1}{4(x-2)^2}$
   c) $\dfrac{1}{2-x}$
   d) $\dfrac{1}{2+x}$

226. If $f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ P & P^2 & P^3 \end{vmatrix}$, here p is a constant, then $\frac{d^3 f(x)}{dx^3}$ is
   a) Proportional to x^2
   b) Proportional to x
   c) Proportional to x^3
   d) A constant

227. If $f(x) = \arctan\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to
   a) 1
   b) -1
   c) $\log 2$
   d) None of these

228. If for all $x, y \in R$, the function f is defined by $f(x) + f(y) + f(x)f(y) = 1$ and $f(x) > 0$. Then,
   a) $f'(x) = 0$ for all $x \in R$
   b) $f'(0) < f'(1)$
   c) $f'(x)$ does not exist
   d) None of these

229. Let $f(x) = e^x$, $g(x) = \sin^{-1} x$ and $h(x) = f[g(x)]$, then $\frac{h'(x)}{h(x)}$ is equal to
   a) $e^{\sin^{-1} x}$
   b) $\dfrac{1}{\sqrt{1-x^2}}$
   c) $\sin^{-1} x$
   d) $\dfrac{1}{(1-x^2)}$

230. If $f(x, y) = \frac{\cos(x-4y)}{\cos(x+4y)}$, then $\left.\frac{\partial f}{\partial x}\right|_{y=\frac{\pi}{2}}$ is equal to
   a) -1
   b) 0
   c) 1
   d) 2

231. If $y = \sin^n x \cos nx$, then $\frac{dy}{dx}$ is
   a) $n\sin^{n-1} x \sin(n+1)x$
   b) $n\sin^{n-1} x \cos(n-1)x$
   c) $n\sin^{n-1} x \cos nx$
   d) $n\sin^{n-1} x \cos(n+1)x$

232. If $2f(x) = f'(x)$ and $f(0) = 3$, then $f(2)$ is equal to
   a) $3e^4$
   b) $3e^2$
   c) e^4
   d) None of these

233. If $y^2 = P(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left[y^3 \frac{d^2 y}{dx^2}\right]$ equals
   a) $P'''(x) + P'x$
   b) $P''(x).P'''(x)$
   c) $P(x).P'''(x)$
   d) None of these

234. If $e^{y+e^{y+\cdots\infty}}$, $x > 0$, then $\frac{dy}{dx}$ is
   a) $\dfrac{x}{1+x}$
   b) $\dfrac{1}{x}$
   c) $\dfrac{1-x}{x}$
   d) $\dfrac{1+x}{x}$

235. The 2nd derivative of $a\sin^3 t$ with respect to $a\cos^3 t$ at $t = \frac{\pi}{4}$ is

a) $\dfrac{4\sqrt{2}}{3a}$ b) 2   c) $\dfrac{1}{12a}$ d) None of these

236. The derivative of $\sin(x^3)$ w.r.t. $\cos(x^3)$ is

a) $-\tan(x^3)$ b) $\tan(x^3)$ c) $-\cot(x^3)$ d) $\cot(x^3)$

237. If $y = \tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{1}{2}$ b) 2   c) -2 d) $-\dfrac{1}{2}$

238. If $y = \tan^{-1}\sqrt{\dfrac{1-\sin x}{1+\sin x}}$, then the value of $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{6}$ is

a) $-\dfrac{1}{2}$ b) $\dfrac{1}{2}$ c) 1   d) -1

239. If $y = \log_{\cos x}\sin x$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{(\cot x \log\cos x + \tan x \log\sin x)}{(\log\cos x)^2}$

b) $\dfrac{(\tan x \log\cos x + \cot x \log\sin x)}{(\log\cos x)^2}$

c) $\dfrac{(\cot x \log\cos x + \tan x \log\sin x)}{(\log\sin x)^2}$

d) None of the above

240. If $y = 2^x \cdot 3^{2x-1}$, then $\dfrac{d^2 y}{dx^2}$ is equal to

a) $(\log 2)(\log 3)$ b) $(\log 18)$ c) $(\log 18^2)\, y^2$ d) $(\log 18)^2\, y$

241. If $y = \sec^{-1}\dfrac{x+1}{x-1} + \sin^{-1}\dfrac{x-1}{x+1}$, then $\dfrac{dy}{dx}$ is

a) 1   b) 0   c) $\dfrac{x-1}{x+1}$ d) $\dfrac{x+1}{x-1}$

242. Let $g(x) = \log f(x)$, where $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = xf(x)$. Then, for $N = 1,2,3,\ldots$, $g''\left(N + \dfrac{1}{2}\right) - g''\left(\dfrac{1}{2}\right)$ is equal to

a) $-4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \cdots + \dfrac{1}{(2N-1)^2}\right\}$

b) $4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \cdots + \dfrac{1}{(2N-1)^2}\right\}$

c) $-4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \cdots + \dfrac{1}{(2N+1)^2}\right\}$

d) $4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \cdots + \dfrac{1}{(2N+1)^2}\right\}$

243. If $y = \log_{x^2+4}(7x^2 - 5x + 1)$, then $\dfrac{dy}{dx}$ is equal to

a) $\log_e(x^2+4) \cdot \left\{\dfrac{14x-5}{7x^2-5x+1} - \dfrac{2\,xy}{x^2+4}\right\}$

b) $\dfrac{1}{\log_e(x^2+4)}\left\{\dfrac{14x-5}{7x^2-5x+1} - \dfrac{2xy}{x^2+4}\right\}$

c) $\log_e(7x^2-5x+1)\left\{\dfrac{2\,x}{x^2+4} - \dfrac{(14x-5)y}{7x^2-5x+1}\right\}$

d) $\dfrac{1}{\log_e(7x^2-5x+1)}\left\{\dfrac{2x}{x^2+4} - \dfrac{(14x-5)y}{7x^2-5x+1}\right\}$

244. If $\sin y = e^{-x\cos y} = e$, then $\dfrac{dy}{dx}$ at $(1, \pi)$ is equal to

a) $\sin y$ b) $-x\cos y$ c) e d) $\sin y - x\cos y$

245. If $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$, then $y'(0)$ is

a) $\dfrac{1}{2}$ b) 0   c) 1   d) -1

246. Differential coefficient of $\sqrt{\sec\sqrt{x}}$ is

a) $\dfrac{1}{4\sqrt{x}}\sec\sqrt{x}\,\sin\sqrt{x}$

b) $\dfrac{1}{4\sqrt{x}}\left(\sec\sqrt{x}\right)^{3/2}.\sin\sqrt{x}$

c) $\dfrac{1}{2}\sqrt{x}\,\sec\sqrt{x}\,\sin\sqrt{x}$

d) $\dfrac{1}{2}\sqrt{x}\left(\sec\sqrt{x}\right)^{3/2}.\sin\sqrt{x}$

247. If $f(x) = \sin x$ and $g(x) = sgn \sin x$, then $g'(1)$ equals

a) 0   b) $-\cos 1$ c) $\cos 1$ d) None of these

248. The derivative of $y = x^{\ln x}$ is

a) $x^{\ln x}\ln x$ b) $x^{\ln x - 1}\ln x$ c) $2x^{\ln x - 1}\ln x$ d) $x^{\ln x - 2}$

249. If $x = e^t \sin t, y = e^t \cos t,$ then $\frac{d^2 y}{dx^2}$ at $x = \pi$ is

a) $2e^\pi$ b) $\frac{1}{2}e^\pi$ c) $\frac{1}{2e^\pi}$ d) $\frac{2}{e^\pi}$

250. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ equals

a) $\sin(\log x) \cdot \dfrac{1}{x \log x}$

b) $\dfrac{12}{(3-2x)^2}\sin\left\{\log\left(\dfrac{2x+3}{3-2x}\right)\right\}$

c) $\sin\left\{\log\left(\dfrac{2x+3}{3-2x}\right)\right\}$

d) None of these

251. If $r = [2\phi + \cos^2(2\phi + \pi/4)]^{1/2}$, then what is the value of the derivative of $dr/d\phi$ at $\phi = \pi/4$?

a) $2\left(\dfrac{1}{\pi+1}\right)^{1/2}$ b) $2\left(\dfrac{2}{\pi+1}\right)^{2}$ c) $\left(\dfrac{2}{\pi+1}\right)^{1/2}$ d) $2\left(\dfrac{2}{\pi+1}\right)^{1/2}$

252. For $|x| < 1$, let $y = 1 + x + x^2 \dots$to ∞, then $\frac{dy}{dx}$ equal to

a) $\dfrac{x}{y}$ b) $\dfrac{x^2}{y^2}$ c) $\dfrac{x}{y^2}$ d) $xy^2 + y$

253. If $f(x + y) = 2f(x)f(y)$, $f'(5) = 1024(\log 2)$ and $f(2) = 8$, then the value of $f'(3)$ is

a) 64 (log 2)   b) 128 (log 2)   c) 256   d) 256 (log 2)

254. The value of differentiation of e^{x^2} with respect to e^{2x-1} at $x = 1$ is

a) e b) 0   c) e^{-1} d) 1

255. Let $x = \log_e t, t > 0$ and $y + 1 = t^2$. Then, $\frac{d^2 x}{dy^2}$ is equal to

a) $4e^{2x}$ b) $-\dfrac{1}{2}e^{-4x}$ c) $-\dfrac{3}{4}e^{5x}$ d) $4e^x$

256. If $y = \cot^{-1}(\cos 2x)^{1/2}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ will be

a) $\left(\dfrac{2}{3}\right)^{1/2}$ b) $\left(\dfrac{1}{3}\right)^{1/2}$ c) $(3)^{1/2}$ d) $(6)^{1/2}$

257. If $P(x)$ is a polynomial such that $P(x^2 + 1) = \{P(x)\}^2 + 1$ and $P(0) = 0$, then $P'(0)$ is equal to

a) -1 b) 0   c) 1   d) None of these

258. If $y = (\cos x^2)^2$, then $\frac{dy}{dx}$ is equal to

a) $-4x\sin 2x^2$ b) $-x\sin x^2$ c) $-2x\sin 2x^2$ d) $-x\cos 2x^2$

259. Let $f(x) = 2^{2x-1}$ and $g(x) = -2^x + 2x\log 2$. Then the set of points satisfying $f'(x) > g'(x)$, is

a) $(0, 1)$ b) $[0, 1)$ c) $(0, \infty)$ d) $[0, \infty)$

260. $\dfrac{d}{dx}\left\{\begin{array}{l}\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) + \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right) \\ \quad - \tan^{-1}\left(\dfrac{4x-4x^3}{1-6x^2+x^4}\right)\end{array}\right\}$ is equal to

a) $\dfrac{1}{\sqrt{1-x^2}}$ b) $-\dfrac{1}{\sqrt{1-x^2}}$ c) $\dfrac{1}{1+x^2}$ d) $-\dfrac{1}{1+x^2}$

261. If $y = \dfrac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{1}{2}\text{cosec}^2\dfrac{x}{2}$ b) $\dfrac{1}{2}\text{cosec}\dfrac{x}{2}$ c) $\dfrac{1}{2}\text{cosec}^2 x$ d) $\text{cosec}^2\dfrac{x}{2}$

262. The value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, where y is given by $y = x^{\sin x} + \sqrt{x}$, is

a) $1 + \dfrac{1}{\sqrt{2\pi}}$ 　　　 b) 1 　　　 c) $\dfrac{1}{\sqrt{2\pi}}$ 　　　 d) $1 - \dfrac{1}{\sqrt{2\pi}}$

263. If $y = \dfrac{3at^2}{1+t^3}, x = \dfrac{3at}{1+t^3}$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{t(2 - t^3)}{(1 - 2t^3)}$ 　　 b) $\dfrac{t(2 + t^3)}{(1 - 2t^3)}$ 　　 c) $\dfrac{t(2 - t^3)}{(1 + 2t^3)}$ 　　 d) $\dfrac{t(2 + t^3)}{(1 + 2t^3)}$

264. If $y = \log_a x + \log_x a + \log_x x + \log_a a$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{1}{x} + x \log a$ 　　 b) $\dfrac{\log a}{x} + \dfrac{x}{\log a}$ 　　 c) $\dfrac{1}{x \log a} + x \log a$ 　　 d) None of these

265. If $8f(x) + 6f\left(\dfrac{1}{x}\right) = x + 5$ and $y = x^2 f(x)$, then the value of $\dfrac{dy}{dx}$ at $x = -1$, is

a) 0 　　　 b) $\dfrac{1}{14}$ 　　　 c) $-\dfrac{1}{14}$ 　　　 d) $\dfrac{1}{7}$

266. If $y = \sqrt{\dfrac{1-x}{1+x}}$, then $(1 - x^2)\dfrac{dy}{dx} + y$ is equal to

a) 1 　　　 b) -1 　　　 c) 2 　　　 d) 0

267. If $x^y = e^{2(x-y)}$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{2(1 + \log x)}{(2 + \log x)^2}$ 　　 b) $\dfrac{1 + \log x}{(2 + \log x)^2}$ 　　 c) $\dfrac{2}{2 + \log x}$ 　　 d) $\dfrac{2(1 - \log x)}{(2 + \log x)^2}$

268. If $x = a(1 + \cos\theta), y = a(\theta + \sin\theta)$, then $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{2}$ is

a) $-\dfrac{1}{a}$ 　　　 b) $\dfrac{1}{a}$ 　　　 c) -1 　　　 d) -2

269. If $y^2 = ax^2 + bx + c$, where a, b, c are constants, then $y^3 \dfrac{d^2y}{dx^2}$ is equal to

a) a constant 　　　　　　　　　　 b) a function of x

c) a function of y 　　　　　　　 d) a function of x and y both

270. If $y = 1 + \dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} + \cdots \infty$ with $|x| > 1$, then $\dfrac{dy}{dx}$ is

a) $\dfrac{x^2}{y^2}$ 　　　 b) $x^2 y^2$ 　　　 c) $\dfrac{y^2}{x^2}$ 　　　 d) $-\dfrac{y^2}{x^2}$

271. If $f(x, y) = 2(x - y)^2 - x^4 - y^4$, then $\left|(f_{xx}f_{yy} - f_{xy}^2)\right|_{0,0}$ is

a) 32 　　　 b) 16 　　　 c) 0 　　　 d) -1

272. If $y = \left[\tan^{-1}\dfrac{1}{1+x+x^2} + \tan^{-1}\dfrac{1}{x^2+3x+3} + \tan^{-1}\dfrac{1}{x^2+5x+7} + \ldots + \text{upto } n \text{ terms}\right]$, then $y'(0)$ is equal to

a) $\dfrac{-1}{(n^2 + 1)}$ 　　 b) $\dfrac{-n^2}{(n^2 + 1)}$ 　　 c) $\dfrac{n^2}{(n^2 + 1)}$ 　　 d) None of these

273. Given that $\dfrac{d}{dx}f(x) = f'(x)$. The relationship $f'(a + b) = f'(a) + f'(b)$ is valid, if $f(x)$ is equal to

a) x 　　　 b) x^2 　　　 c) x^3 　　　 d) x^4

274. nth derivative of $(x + 1)^n$ is equal to

a) $(n - 1)!$ 　　 b) $(n + 1)!$ 　　 c) $n!$ 　　 d) $n[(n + 1)]^{n-1}$

275. If $y = 2^x \cdot 3^{2x-1}$, then $\dfrac{dy}{dx}$ is equal to

a) $(\log 2)(\log 3)$ 　　 b) $(\log 18)$ 　　 c) $(\log 18^2)y^2$ 　　 d) $(\log 18)y$

276. If $y = x^2 + \dfrac{1}{x^2 + \dfrac{1}{x^2 + \dfrac{1}{x^2 + \cdots \infty}}}$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{2xy}{2y - x^2}$ 　　 b) $\dfrac{xy}{y + x^2}$ 　　 c) $\dfrac{xy}{y - x^2}$ 　　 d) $\dfrac{2x}{2 + \dfrac{x^2}{y}}$

277. If $y = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$, then $\dfrac{dy}{dx}$ is equal to

a) $\text{sech}^2 x$ 　　 b) $\text{cosech}^2 x$ 　　 c) $-\text{sech}^2 x$ 　　 d) $-\text{cosech}^2 x$

278. If $f(x) = |x - 2|$ and $g(x) = f(f(x))$, then for $x > 20$, $g'(x)$ is equal to

a) -1 b) 0 c) 1 d) 2

279. If $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2$ and $y = x f(x)$, then $\left(\frac{dy}{dx}\right)_{x=1}$ is equal to

    a) 14 b) $7/8$ c) 1 d) None of these

280. The differential coefficient of $f(\log x)$, where $f(x) = \log x$ is

    a) $\frac{x}{\log x}$ b) $(x \log x)^{-1}$ c) $\frac{\log x}{x}$ d) None of these

281. Let $f(x) = \frac{x^2}{1-x^2}, x \neq 0, \pm 1$, then derivative of $f(x)$ with respect to x is

    a) $\frac{2x}{(1-x^2)^2}$ b) $\frac{1}{(2+x^2)^3}$ c) $\frac{1}{(1-x^2)^2}$ d) $\frac{1}{(2-x^2)^2}$

282. If $f(x) = |x|^3$, then $f'(0)$ equal to

    a) 0 b) $1/2$ c) -1 d) $-\frac{1}{2}$

283. The derivative of $\log |x|$ is

    a) $\frac{1}{x}, x > 0$ b) $\frac{1}{|x|}, x \neq 0$ c) $\frac{1}{x}, x \neq 0$ d) None of these

284. If $f(x) = \tan^{-1}\left\{\frac{\log\left(\frac{e}{x^2}\right)}{\log(e x^2)}\right\} + \tan^{-1}\left(\frac{3+2 \log x}{1-6 \log x}\right)$, then $\frac{d^n y}{dx^n}$ is

    a) $\tan^{-1}\{(\log x)^n\}$ b) 0 c) $1/2$ d) None of these

285. If $f(x) = x + 2$, then $f'\big(f(x)\big)$ at $x = 4$, is

    a) 8 b) 1 c) 4 d) 5

286. If $f(x) - \log_e(\log_e x)$, then $f'(x)$ at $x = e$, is

    a) 0 b) 1 c) $\frac{1}{e}$ d) $\frac{e}{2}$

287. If $\sin(x + y) + \cos(x + y) = \log(x + y)$, then $\frac{d^2 y}{dx^2}$ is

    a) $\frac{-y}{x}$ b) 0 c) -1 d) 1

288. $10^{-x \tan x}\left[\frac{d}{dx}(10^{x \tan x})\right]$ is equal to

    a) $\tan x + x \sec^2 x$ b) $\ln 10 \,(\tan x + x \sec^2 x)$

    c) $\ln 10 \left(\tan x + \frac{x}{\cos^2 x} + \tan x \sec x\right)$ d) $x \tan x \ln 10$

289. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$, then $\frac{d^2 y}{dx^2}$ is equal to

    a) $-x$ b) x c) y d) $-y$

290. Differential coefficient of $\sec^{-1}\frac{1}{2x^2-1}$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is equal to

    a) 2 b) 4 c) 6 d) 1

291. If $y = \cos 2 x \cos 3 x$, then y_n is equal to

    a) $6^n \cos\left(2 x + \frac{n\pi}{2}\right) \cos\left(3 x + \frac{n\pi}{2}\right)$

    b) $6^n \cos\left(2 x + \frac{n\pi}{2}\right) \cos\left(\frac{3 x + n\pi}{2}\right)$

    c) $\frac{1}{2}\left\{5^n \sin\left(5x + \frac{n\pi}{2}\right) + \sin\left(x + \frac{\pi}{2}\right)\right\}$

    d) None of these

292. If $f(x) = \log_{x^2}(\log_e x)$, then $f'(x)$ at $x = e$ is

    a) 1 b) $\frac{1}{e}$ c) $\frac{1}{2e}$ d) 0

293. Let $f x) = \sin x, g(x) = x^2$ and $h(x) = \log_e x$. If $F(x) = (h o g o f)(x)$, then $F''(x)$ is equal to

    a) $a \operatorname{cosec}^3 x$ b) $2 \cot x^2 - 4x^2 \operatorname{cosec}^2 x^2$ c) $2x \cot x^2$ d) $-2 \operatorname{cosec}^2 x$

294. $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right), \; y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right) \Rightarrow \frac{dy}{dx}$ is equal to

    a) 0 b) $\tan t$ c) 1 d) $\sin t \cos t$

295. If $y = \tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is equal to

 a) 2   b) 1   c) 0   d) -1

296. $\frac{d}{dx}\left[\sin^2\cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$ equals

 a) -1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1

297. If $f(x) = \left\{\frac{\pi}{2}[x] - x^5\right\}, 1 < x < 2$ and $[\cdot]$ denotes the greatest integer function, then $f'\left(\sqrt[5]{\frac{\pi}{2}}\right)$ is equal to

 a) 0   b) $5(\pi/2)^{4/5}$ c) $-5(\pi/2)^{4/5}$ d) None of these

298. Let f be twice differentiable function such that $f''(x) = -f(x)$, and $f'(x) = g(x)$, $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to

 a) 22   b) 11   c) 0   d) 20

299. $f(x) = \begin{vmatrix} x^3 & x^4 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$, here p is a constant, then $\frac{d^4 f(x)}{dx^4}$ is

 a) Proportional to x^2 b) Proportional to x c) Proportional to x^3 d) A constant

300. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is

 a) e^x b) $-\frac{e^x}{(1+e^x)^3}$ c) $-\frac{e^x}{(1+e^x)^2}$ d) $\frac{1}{(1+e^x)^2}$

301. If $y = f\left(\frac{2x+3}{3-2x}\right)$ and $f(x) = \sin(\log x)$, then $\frac{dy}{dx}$ is equal to

 a) $\frac{12}{9-4x^2}\cos\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$
 b) $\frac{12}{4x^2-9}\cos\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$
 c) $\frac{12}{9-4x^2}\cos\left\{\log\left(\frac{3-2x}{2x+3}\right)\right\}$
 d) $\frac{12}{9-4x^2}\cos\left\{\log\left(\frac{2x+3}{2x-3}\right)\right\}$

302. If $y = \cos^{-1}\left(\frac{1-\log x}{1+\log x}\right)$, then $\frac{dy}{dx}$ at $x = e$ is

 a) $-\frac{1}{e}$ b) $-\frac{1}{2e}$ c) $\frac{1}{2e}$ d) $\frac{1}{e}$

303. If $x^y = y^x$, then $x(x - y\log x)\frac{dy}{dx}$ is equal to

 a) $y(y - x\log y)$ b) $y(y + x\log y)$ c) $x(x + y\log x)$ d) $x(y - x\log y)$

304. If variables x and y are related by the equation $x = \int_0^y \frac{1}{\sqrt{1+9u^2}} du$, then $\frac{dy}{dx}$ is equal to

 a) $\frac{1}{\sqrt{1+9y^2}}$ b) $\sqrt{1+9y^2}$ c) $1+9y^2$ d) $\frac{1}{1+9y^2}$

305. If $y = \sec^{-1}[\operatorname{cosec} x] + \operatorname{cosec}^{-1}[\sec x] + \sin^{-1}[\cos x] + \cos^{-1}[\sin x]$, then $\frac{dy}{dx}$ is equal

 To

 a) 0   b) 2   c) -2 d) -4

306. The value of $\frac{d}{dx}(|x - 1| + |x - 5|)$ at $x = 3$ is

 a) -2 b) 0   c) 2   d) 4

307. If $f(x) = \log_e\left\{\frac{u(x)}{v(x)}\right\}, u(1) = v(1)$ and $u'(1) = v'(1) = 2$, then $f'(1)$ is equal to

 a) 0   b) 1   c) -1 d) None of these

308. $\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\right\}$ equals

 a) -1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1

309. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

a) $-\dfrac{y}{x}$ b) $\dfrac{y}{x}$ c) $-\dfrac{x}{y}$ d) $\dfrac{x}{y}$

310. If $x = y\sqrt{1 - y^2}$, then $\dfrac{dy}{dx}$ is equal to

     a) x b) $\dfrac{\sqrt{1 - y^2}}{1 + 2y^2}$ c) 0 d) $\dfrac{\sqrt{1 - y^2}}{1 - 2y^2}$

311. If $f(x) = (x + 1)\tan^{-1}(e^{-2x})$, then $f'(0)$ is

     a) $\dfrac{\pi}{2} + 1$ b) $\dfrac{\pi}{4} - 1$ c) $\dfrac{\pi}{6} + 5$ d) $\dfrac{\pi}{4} + 1$

312. If $y = 5^x \, x^5$, then $\dfrac{dy}{dx}$ is

     a) $5^x(x^5 \log 5 - 5x^4)$ b) $x^5 \log 5 - 5x^4$ c) $x^5 \log 5 + 5x^4$ d) $5^x(x^5 \log 5 + 5x^4)$

313. If $y = a \cos(\log x) + b \sin(\log x)$ where a, b are parameters, then $x^2 y'' + xy'$ is equal to

     a) y b) $-y$ c) $2y$ d) $-2y$

314. If $y = \sin^{-1}\left(\dfrac{\sin\alpha \sin x}{1 - \cos\alpha \sin x}\right)$, then $y'(0)$ is

     a) 1 b) $2\tan\alpha$ c) $(1/2)\tan\alpha$ d) $\sin\alpha$

315. If $f(x) = |x - 2| + |x + 1| - x$, then $f'(-10)$ is equal to

     a) -3 b) -2 c) -1 d) 0

316. If $f: R \to R$ is an even function which us twice differentiable on R and $f''(\pi) = 1$, then $f''(-\pi)$

     a) -1 b) 0 c) 1 d) 2

317. $\dfrac{d}{dx}\left[\log_e e^{\sin(x^2)}\right]$ is equal to

     a) $2\cos(x^2)$ b) $2\cos x$ c) $2x.\cos x$ d) $2x\cos(x^2)$

318. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$, then $f^2(4) + g^2(4)$ is

     a) 8 b) 16 c) 32 d) 64

319. $z = \tan(y + ax) + \sqrt{y - ax} \Rightarrow z_{xx} - a^2 z_{yy}$ is equal to

     a) 0 b) 2 c) $z_x + z_y$ d) $z_x z_y$

320. $\dfrac{d}{dx}(\log x)^4$ is equal to

     a) $4\log x^3$ b) $4(\log x)^3$ c) $\dfrac{4(\log x)^3}{x}$ d) $\dfrac{(4\log x)^3}{x}$

321. If $\sin^{-1}\left(\dfrac{x^2 - y^2}{x^2 + y^2}\right) = \log a$, then $\dfrac{dy}{dx}$ equals

     a) $\dfrac{x}{y}$ b) $\dfrac{y}{x^2}$ c) $\dfrac{x^2 - y^2}{x^2 + y^2}$ d) $\dfrac{y}{x}$

322. The differential coefficient of $a^{\log_{10}\cosec^{-1}x}$, is

     a) $\dfrac{a^{\log_{10}(\cosec^{-1}x)}}{\cosec^{-1}x}\dfrac{1}{x\sqrt{x^2 - 1}}\log_{10} a$

     b) $-\dfrac{a^{\log_{10}(\cosec^{-1}x)}}{\cosec^{-1}x}\cdot\dfrac{1}{|x|\sqrt{x^2 - 1}}\log_{10} a$

     c) $\dfrac{-a^{\log_{10}(\cosec^{-1}x)}}{\cosec^{-1}x}\cdot\dfrac{1}{|x|\sqrt{x^2 - 1}}\log_a 10$

     d) $\dfrac{a^{\log_{10}\cosec^{-1}x}}{\cosec^{-1}x}\cdot\dfrac{1}{|x|\sqrt{x^2 - 1}}\log_a 10$

323. If $f(x) = \sqrt{1 + \cos^2(x)^2}$, then the value of $f'\left(\dfrac{\sqrt{\pi}}{2}\right)$ is

     a) $\dfrac{\sqrt{\pi}}{6}$ b) $-\sqrt{\dfrac{\pi}{6}}$ c) $\dfrac{1}{\sqrt{6}}$ d) $\dfrac{\pi}{\sqrt{6}}$

324. $\dfrac{d^n}{dx^n}(\log x)$ is equal to

a) $\dfrac{(n-1)!}{x^n}$ b) $\dfrac{n!}{x^n}$ c) $\dfrac{(n-2)!}{x^n}$ d) $(-1)^{n-1}\dfrac{(n-1)!}{x^n}$

325. $\dfrac{d^2x}{dy^2}$ equals

a) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$ b) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ c) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$ d) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

326. The second order derivative of $a\sin^3 t$ with respect to $a\cos^3 t$ at $t=\dfrac{\pi}{4}$ is

a) 2   b) $\dfrac{1}{12a}$ c) $\dfrac{4\sqrt{2}}{3a}$ d) $\dfrac{3a}{4\sqrt{2}}$

327. If $\dfrac{d^2x}{dy^2}\left(\dfrac{dy}{dx}\right)^3 + \dfrac{d^2y}{dx^2} = k$, then k is equal to

a) 0   b) 1   c) 2   d) None of these

328. If $e^y + xy = e$, then the value of $\dfrac{d^2y}{dx^2}$ for $x = 0$, is

a) $1/e$ b) $1/e^2$ c) $1/e^3$ d) e

329. $\dfrac{d}{dx}\sin^{-1}\left(x\sqrt{1-x}+\sqrt{x}\sqrt{1-x^2}\right)$ is equal to

a) $-\dfrac{1}{2x\sqrt{1-x}}-\dfrac{1}{\sqrt{1-x^2}}$ b) $\dfrac{1}{2\sqrt{x}\sqrt{1-x^2}}-\dfrac{1}{\sqrt{1-x^2}}$

c) $\dfrac{1}{2\sqrt{x}\sqrt{1-x}}+\dfrac{1}{\sqrt{1-x^2}}$ d) $-\dfrac{1}{2\sqrt{x}\sqrt{1-x}}+\dfrac{1}{\sqrt{1-x^2}}$

330. If $f(x) = \log_x(\log_e x)$, then $f'(x)$ at $x = e$ is equal to

a) 1   b) 2   c) 0   d) $\dfrac{1}{e}$

331. If $y = \log_u|\cos 4x| + |\sin x|$, where $u = \sec 2x$, then $\dfrac{dy}{dx}$ at $x = -\dfrac{\pi}{6}$ is equal to

a) $\dfrac{-6\sqrt{3}}{\log_e 2}-\dfrac{\sqrt{3}}{2}$ b) $\dfrac{-6\sqrt{3}}{\log_e 2}+\dfrac{\sqrt{3}}{2}$ c) $\dfrac{6\sqrt{3}}{\log_e 2}+\dfrac{\sqrt{3}}{2}$ d) None of these

332. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a(x^3-y^3)$ and $\dfrac{dy}{dx}=f(x,y)\sqrt{\dfrac{1-y^6}{1-x^6}}$, then $f(x,y)=$

a) $\dfrac{y}{x}$ b) $\dfrac{x^2}{y^2}$ c) $\dfrac{2y^2}{x^2}$ d) $\dfrac{y^2}{x^2}$

333. If $y = \log^n x$, where $\log^n$ means $\log\log\log\ldots$ (repeated n times), then

$x\log x\log^2 x\log^3 x\ldots\log^{n-1}x\log^n x\dfrac{dy}{dx}$ is equal to

a) $\log x$ b) x c) $\dfrac{1}{\log x}$ d) $\log^n x$

334. If $x^2+y^2=t+\dfrac{1}{t}$ and $x^4+y^4=t^2+\dfrac{1}{t^2}$, then $\dfrac{dy}{dx}$ is equal to

a) $\dfrac{y}{x}$ b) $-\dfrac{y}{x}$ c) $\dfrac{x}{y}$ d) $-\dfrac{x}{y}$

335. If $y = \sqrt{x\log_e x}$, then $\dfrac{dy}{dx}$ at $x = e$ is

a) $\dfrac{1}{e}$ b) $\dfrac{1}{\sqrt{e}}$ c) $\sqrt{e}$ d) e^2

336. If $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, then $\dfrac{dy}{dx}$ equals

a) $\dfrac{1}{\sqrt{1-x^4}}$ b) $-\dfrac{1}{\sqrt{1-x^4}}$ c) $\dfrac{x}{\sqrt{1-x^4}}$ d) $-\dfrac{x}{\sqrt{1-x^4}}$

337. If $f'(x) = \sin(\log x)$ and $y = f\left(\dfrac{2x+3}{3-2x}\right)$, then $\dfrac{dy}{dx}$ at $x = 1$ is equal to

a) $6\sin\log(5)$ b) $5\sin\log(6)$ c) $12\sin\log(5)$ d) $5\sin\log(12)$

338. If $x^y = e^{x-y}$, $x > 0$ then the value of $\dfrac{dy}{dx}$ at $(1,1)$ is

a) 0   b) $\frac{1}{2}$ c) 1   d) 2

339. If $y = \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ is equal to

a) $-\frac{1}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{3}{\sqrt{1-x^2}}$ d) $\frac{x}{\sqrt{1-x^2}}$

340. If $y = e^x \cdot e^{x^2} \cdot e^{x^3} \cdots e^{x^n} \cdots$, for $0 < x < 1$, then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is

a) e b) $4e$ c) $2e$ d) $3e$

341. If $y = \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{\sqrt{1-x^2}}$ b) $\frac{-1}{\sqrt{1-x^2}}$ c) $\frac{3}{\sqrt{1-x^2}}$ d) $\frac{1}{\sqrt{1+x^2}}$

342. For $0 < x < 2$, $\frac{d}{dx}\left(\tan^{-1}\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}}\right)$ is equal to

a) $-1/4$ b) $1/4$ c) $-1/2$ d) $1/2$

343. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

a) -1 b) 1   c) 2   d) 4

344. Let $f(x)$ be a polynomial function of the second degree. If $f(1) = f(-1)$ and a_1, a_2, a_3 are in AP, then $f'(a_1), f'(a_2), f'(a_3)$ are in

a) AP   b) GP   c) HP   d) None of these

345. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$ is

a) 1   b) $\frac{3}{2}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$

346. If $y = \log_{\sin x} \cos x$, then $\frac{dy}{dx}$ is equal to

a) $\frac{\tan x \log(\sin x) - \cot x \log\cos x}{(\log\sin x)^2}$ b) $\frac{-\tan x \log\sin x - \cot x \log\cos x}{(\log\sin x)^2}$

c) $\frac{-\tan x \log(\sin x) + \cot x \log(\cos x)}{(\log\sin x)^2}$ d) $\frac{\tan x \log(\sin x) + \cot\log(\cos x)}{[\log(\sin x)]^2}$

347. If $y = \tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is

a) 2   b) 1   c) 0   d) -1

348. If $y = c\, e^{x/(x-a)}$, then $\frac{dy}{dx}$ equals

a) $a(x-a)^2$ b) $-\frac{ay}{(x-a)^2}$ c) $a^2(x-a)^2$ d) $a(x-a)$

349. If $y = \sin^{-1}\sqrt{1-x}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{\sqrt{1-x}}$ b) $\frac{-1}{2\sqrt{1-x}}$ c) $\frac{1}{\sqrt{x}}$ d) $\frac{-1}{2\sqrt{x}\sqrt{1-x}}$

350. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is

a) $\frac{-2}{1+x^2}$ for all x b) $\frac{-2}{1+x^2}$ for all $|x| > 1$ c) $\frac{2}{1+x^2}$ for all $|x| > 1$ d) None of these

351. If $y = ae^x + be^{-x} + c$ where a, b, c are parameters, then y' is equal to

a) y b) y' c) 0   d) y''

DIFFERENTITATION

1) b	2) b	3) c	4) d	153) d	154) c	155) c	156) d
5) c	6) d	7) b	8) d	157) b	158) b	159) a	160) b
9) c	10) c	11) b	12) d	161) b	162) d	163) b	164) c
13) d	14) c	15) b	16) a	165) c	166) a	167) c	168) b
17) b	18) a	19) c	20) d	169) b	170) d	171) d	172) a
21) b	22) d	23) c	24) c	173) c	174) b	175) a	176) b
25) b	26) b	27) d	28) d	177) b	178) b	179) b	180) c
29) c	30) d	31) d	32) a	181) a	182) b	183) b	184) b
33) c	34) c	35) c	36) d	185) a	186) c	187) b	188) c
37) b	38) d	39) c	40) b	189) b	190) c	191) a	192) d
41) a	42) c	43) c	44) a	193) b	194) c	195) c	196) b
45) a	46) b	47) a	48) c	197) c	198) b	199) d	200) c
49) c	50) a	51) b	52) c	201) a	202) b	203) a	204) d
53) b	54) b	55) d	56) c	205) d	206) d	207) d	208) c
57) d	58) b	59) b	60) c	209) d	210) a	211) b	212) a
61) b	62) b	63) a	64) c	213) c	214) c	215) c	216) d
65) b	66) b	67) c	68) a	217) b	218) d	219) c	220) b
69) d	70) c	71) b	72) a	221) a	222) c	223) b	224) c
73) a	74) a	75) a	76) d	225) a	226) d	227) d	228) a
77) d	78) c	79) b	80) b	229) b	230) b	231) d	232) a
81) a	82) c	83) d	84) d	233) c	234) c	235) a	236) c
85) c	86) d	87) b	88) c	237) d	238) a	239) a	240) d
89) d	90) b	91) c	92) a	241) b	242) a	243) b	244) c
93) a	94) a	95) a	96) b	245) a	246) b	247) c	248) c
97) d	98) d	99) c	100) a	249) d	250) b	251) d	252) d
101) c	102) b	103) c	104) c	253) a	254) d	255) b	256) a
105) c	106) c	107) b	108) a	257) c	258) c	259) c	260) c
109) c	110) d	111) c	112) c	261) a	262) a	263) a	264) d
113) c	114) a	115) c	116) a	265) c	266) d	267) a	268) a
117) b	118) b	119) b	120) b	269) a	270) d	271) c	272) c
121) d	122) a	123) c	124) a	273) b	274) c	275) d	276) a
125) a	126) b	127) d	128) b	277) d	278) c	279) b	280) b
129) d	130) d	131) b	132) d	281) a	282) a	283) c	284) b
133) d	134) b	135) c	136) a	285) b	286) c	287) b	288) b
137) b	138) b	139) d	140) a	289) c	290) b	291) d	292) c
141) b	142) a	143) c	144) b	293) d	294) c	295) c	296) b
145) a	146) b	147) d	148) b	297) c	298) b	299) d	300) b
149) b	150) a	151) a	152) d	301) a	302) c	303) a	304) b

305) d	306) b	307) a	308) b	333) d	334) b	335) b	336) d
309) c	310) b	311) b	312) d	337) c	338) a	339) b	340) b
313) b	314) d	315) a	316) c	341) a	342) a	343) b	344) a
317) d	318) c	319) a	320) c	345) c	346) b	347) c	348) b
321) d	322) b	323) b	324) d	349) d	350) d	351) b	
325) d	326) c	327) a	328) b				
329) c	330) d	331) a	332) b				

DIFFERENTITATION

: HINTS AND SOLUTIONS :

1 (b)

$f(x^3) = 4x^4, \forall x > 0$

Let $x^3 = t \Rightarrow x = t^{1/3}$

$\therefore f(t) = 4t^{4/3}$

On differentiating w.r.t. t, we get

$f'(t) = 4 \cdot \dfrac{4}{3}(t)^{1/3}$

$\therefore f'(x^3) = \dfrac{16}{3}(x^3)^{1/3} = \dfrac{16}{3}x$

$\therefore f'(8) = f'(2^3) = \dfrac{16}{3} \times 2 = \dfrac{32}{3}$

2 (b)

$\because y = x - x^2$

$\therefore \dfrac{dy}{dx} = 1 - 2x$

Now,

$\dfrac{d(y^2)}{d(x^2)} = \dfrac{\frac{d(y^2)}{dx}}{\frac{d(x^2)}{dx}}$

$= \dfrac{2y\frac{dy}{dx}}{2x}$

$= \dfrac{y}{x}(1 - 2x)$

$= (1 - x)(1 - 2x)$

$= 1 - 3x + 2x^2$

$= 2x^2 - 3x + 1$

3 (c)

$\because y = e^{\frac{1}{2}\log(1+\tan^2 x)}$

$= (\sec^2 x)^{1/2} = \sec x$

$\therefore \dfrac{dy}{dx} = \sec x \tan x$

4 (d)

Given, $y = 1 + x + x^2 + x^3 + \ldots \infty$

Since, $|x| < 1$

$\therefore y = \dfrac{1}{(1-x)} = (1-x)^{-1}$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{(1-x)^2}(-1) = \dfrac{1}{(1-x)^2}$

$\therefore \dfrac{dy}{dx} - y = \dfrac{1}{(1-x)^2} - \dfrac{1}{(1-x)} = \dfrac{x}{(1-x)^2}$

$\Rightarrow \dfrac{dy}{dx} - y = xy^2$

5 (c)

$f(x) = x^n \Rightarrow f(1) = 1$

$f'(x) = nx^{n-1} \Rightarrow f'(1) = n$

$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$

$\ldots \ldots \ldots \ldots \ldots \ldots \ldots$

$\ldots \ldots \ldots \ldots \ldots \ldots \ldots$

$f^n(x) = n(n-1)(n-2)\ldots 2.1$

$\Rightarrow f^n(1) = n(n-1)(n-2)\ldots 2.1$

Now, $f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + \ldots + \dfrac{(-1)^n f^n(1)}{n!}$

$= 1 - \dfrac{n}{1!} + \dfrac{n(n-1)}{2!}$

$- \dfrac{n(n-1)(n-2)}{3!} + \ldots + \dfrac{(-1)^n n(n-1)(n-2)\ldots}{n!}$

$= (1-1)^n = 0$

6 (d)

Let $y = 2\cos x \cos 3x$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = 2\cos x(-3\sin 3x) + 2\cos 3x(-\sin x)$

$= -3(\sin 4x + \sin 2x) + (-1)[\sin 4x + \sin(-2x)]$

Again differentiating w.r.t. x, we get

$\dfrac{d^2 y}{dx^2} = -3(4\cos 4x$

$\qquad + 2\cos 2x)$

$\qquad - 1(4\cos 4x - 2\cos 2x)$

$= -16\cos 4x - 4\cos 2x = -4(\cos 2x + 4\cos 4x)$

$= -2^2(\cos 2x + 2^2\cos 4x)$

7 (b)

We have,

$y = x^{x^{x^{x^{\cdots \infty}}}}$

$\Rightarrow y = x^y$

$\Rightarrow y = e^{y \log x}$

$\Rightarrow \dfrac{dy}{dx} = e^{y \log x} \dfrac{d}{dy}(y \log x)$

$\Rightarrow \dfrac{dy}{dx} = x^y \left(\dfrac{dy}{dx} \log x + \dfrac{y}{x}\right)$

$\Rightarrow \dfrac{dy}{dx}(1 - y \log x) = \dfrac{y^2}{x} \quad [\because x^y = y]$

$\Rightarrow x(1 - y \log x)\dfrac{dy}{dx} = y^2$

8 (d)

Given, $f(x) = \dfrac{g(x) + g(-x)}{2} + \dfrac{2}{[h(x) + h(-x)]^{-1}}$

$\Rightarrow (x) = \dfrac{g(x) + g(-x)}{2} + 2[h(x) + h(-x)]$

On differentiating w.r.t. x, we get

$f'(x) = \dfrac{g'(x) - g'(-x)}{2} + 2[h'(x) - h'(-x)]$

$\therefore f'(0) = \dfrac{g'(0) - g'(0)}{2} + 2[h'(0) - h'(0)]$

$= 0$

9 (c)

Let $y = \dfrac{d}{dx}[\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})]$

Put $x = \sin\alpha$ and $\sqrt{x} = \sin\beta$

$\therefore y = \dfrac{d}{dx}[\sin^{-1}(\sin\alpha\sqrt{1 - \sin^2\beta}$

$\qquad\qquad - \sin\beta\sqrt{1 - \sin^2\alpha})]$

$= \dfrac{d}{dx}[\sin^{-1}\sin(\alpha - \beta)] = \dfrac{d}{dx}(\alpha - \beta)$

$= \dfrac{d}{dx}[\sin^{-1}x - \sin^{-1}\sqrt{x}]$

$= \dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{2\sqrt{x}\sqrt{1-x}}$

$= \dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{2\sqrt{x(1-x)}}$

11 (b)

$z = \log(\tan x + \tan y)$

On differentiating partially w.r.t. x and y, we get

$\dfrac{\partial z}{\partial x} = \dfrac{\sec^2 x}{\tan x + \tan y}$ and $\dfrac{\partial z}{\partial y} = \dfrac{\sec^2 y}{\tan x + \tan y}$

Now, $\sin 2x \dfrac{\partial z}{\partial x} + \sin 2y \dfrac{\partial z}{\partial y}$

$= \dfrac{\sin 2x \sec^2 x + \sin 2y \sec^2 y}{\tan x + \tan y}$

$= \dfrac{2[\tan x + \tan y]}{\tan x + \tan y} = 2$

12 (d)

Let $u = \sin^{-1}\left(\dfrac{1}{1 - 2x^2}\right)$, $v = \sin^{-1}(3x - 4x^3)$

Put $x = \sin\theta$, we get

$u = \sec^{-1}(\sec 2\theta)$, $v = \sin^{-1}(\sin 3\theta)$

$\Rightarrow \quad u = 2\theta, \quad v = 3\theta \quad \Rightarrow \quad u$

$\qquad\qquad = 2\sin^{-1}x, \qquad v$

$\qquad\qquad = 3\sin^{-1}x$

$\Rightarrow \dfrac{u}{v} = \dfrac{2}{3} \Rightarrow u = \dfrac{2}{3}v$

$\Rightarrow \dfrac{du}{dv} = \dfrac{2}{3}$

13 (d)

$f(x, y) = \sin u = \dfrac{x^2 y^2}{x + y}$

Here, $f(x)$ is a homogenous function of degree 3.

By Euler theorem,

$x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = 3f$

$x\dfrac{\partial \sin u}{\partial x} + y\dfrac{\partial \sin u}{\partial y} = 3\sin u$

$\Rightarrow x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3\tan u$

14 (c)

$y = \left(1 + \dfrac{1}{x}\right)\left(1 + \dfrac{2}{x}\right)\left(1 + \dfrac{3}{x}\right)\cdots\left(1 + \dfrac{n}{x}\right)$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \left(-\dfrac{1}{x^2}\right)\left(1 + \dfrac{2}{x}\right)\left(1 + \dfrac{3}{x}\right)\cdots\left(1 + \dfrac{n}{x}\right)$

$+ \left(1 + \dfrac{1}{x}\right)\left(-\dfrac{2}{x^2}\right)\left(1 + \dfrac{3}{x}\right)\cdots\left(1 + \dfrac{n}{x}\right)$

$+ \cdots + \left(1 + \dfrac{1}{x}\right)\left(1 + \dfrac{2}{x}\right)\left(1 + \dfrac{3}{x}\right)\cdots\left(-\dfrac{n}{x^2}\right)$

$\therefore \left.\dfrac{dy}{dx}\right|_{x=-1} = (-1)(-1)(-2)(-3)\cdots(1 - n)$

$= (-1)^n(1)(2)(3)\cdots(n - 1)$

$= (-1)^n(n - 1)!$

15 (b)

Given, $y = a\sin^3\theta$ and $x = a\cos^3\theta$

On differentiating w.r.t. θ, we get

$\dfrac{dy}{d\theta} = 3a\sin^2\theta \cos\theta$

and $\dfrac{dx}{d\theta} = -3a\cos^2\theta \sin\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = -\dfrac{3a\sin^2\theta \cos\theta}{3a\cos^2\theta \sin\theta} = -\tan\theta$

At $\theta = \dfrac{\pi}{3}, \dfrac{dy}{dx} = -\tan\dfrac{\pi}{3} = -\sqrt{3}$

16 (a)

Since, to find $\dfrac{dy}{dx}$ at $x = 0$

$\therefore$ At $x = 0, \log(y + 0) = 0 \quad \Rightarrow \quad y = 1$

$\therefore$ To find $\dfrac{dy}{dx}$ at $(0, 1)$

On differentiating given equation w.r.t. x, we get

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) = 2x\frac{dy}{dx} + 2y.1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y(x+y)-1}{1-2(x+y)x}$$

$$\therefore \left(\frac{dy}{dx}\right)_{(0,1)} = 1$$

17 (b)

Since $f(x)$ is a polynomial of degree n satisfying $f(x) = f(\alpha - x)$. Therefore,

$$f(x) = a_0\{x^n + (\alpha - x)^n\}$$
$$+ a_1\{x^{n-1} + (\alpha - x)^{n-1}\} + \cdots +$$
$$\ldots a_{n-1}\{x + (\alpha - x)\} + a_n \text{ where } a_0 \neq 0$$

Clearly, $f'(x)$ is a polynomial of degree $(n-1)$

18 (a)

$$f(x) = \log_5(\log_7 x)$$

$$\Rightarrow f(x) = \log_5\left(\frac{\log_e x}{\log_e 7}\right)$$

$$\Rightarrow f(x) = \log_5 \log_e x - \log_5 \log_e 7$$

$$\Rightarrow f(x) = \frac{\log_e \log_e x}{\log_e 5} - \log_5 \log_e 7$$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{1}{x \log_e x \log_e 5} - 0$$

$$= \frac{1}{\frac{x \log_e x}{\log_e 7} \log_e 7 . \log_e 5}$$

$$= \frac{1}{x \log_7 x \log 7 . \log 5}$$

19 (c)

We have,

$$\frac{dy}{dx} = \sqrt{1 + 9y^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{9y}{\sqrt{1+9y^2}}\frac{dy}{dx} = \frac{9y}{\sqrt{1+9y^2}}\sqrt{1+9y^2}$$
$$= 9y$$

20 (d)

Given, $u = x^2 + y^2, x = s + 3t, y = 2s - t$

Now, $\frac{dx}{ds} = 1, \frac{dy}{ds} = 2 \ldots$ (i)

$$\frac{d^2x}{ds^2} = 0, \frac{d^2y}{ds^2} = 0 \ldots \text{(ii)}$$

Now, $u = x^2 + y^2$

$$\frac{du}{ds} = 2x\frac{dx}{ds} + 2y\frac{dy}{ds}$$

$$\frac{d^2u}{ds^2} = 2\left(\frac{dx}{ds}\right)^2 + 2x\frac{d^2x}{ds^2} + 2\left(\frac{dy}{ds}\right)^2 + 2y\left(\frac{d^2y}{ds^2}\right)$$

$$\Rightarrow \frac{d^2u}{ds^2} = 2(1)^2 + 2x(0) + 2(2)^2 + 2y(0) = 2 + 8$$
$$= 10$$

21 (b)

Given, $f(x) = \sqrt{1 + \cos^2(x^2)}$

Now, $f'(x) = \frac{-2\sin x^2 \cos x^2}{\sqrt{1+\cos^2 x^2}}(x) = \frac{-\sin 2x^2}{\sqrt{1+\cos^2 x^2}}(x)$

$$\Rightarrow f'\left(\frac{\pi}{2}\right) = \frac{\sqrt{\pi}}{2}\frac{\sin 2\left(\frac{\pi}{4}\right)}{\sqrt{1+\left(\cos\frac{\pi}{4}\right)^2}} = -\sqrt{\frac{\pi}{6}}$$

22 (d)

Given $y = 2^{\log x}$

$$\Rightarrow \frac{dy}{dx} = 2^{\log x}.\log_e 2.\frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^{\log x}.\log_e 2}{x}$$

23 (c)

Let $y = \left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$

$$= \log e^x + \log\left(\frac{x-2}{x+2}\right)^{3/4}$$

$$\Rightarrow y = x + \frac{3}{4}[\log(x-2) - \log(x+2)]$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[x + \frac{3}{4}\{\log(x-2) - \log(x+2)\}\right]$$

$$= 1 + \frac{3}{4}\left[\frac{1}{x-2} - \frac{1}{x+2}\right] = 1 + \frac{3}{(x^2-4)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2-1}{x^2-4}$$

24 (c)

Let the even function be

$$f(x) = \cos x$$

$$\Rightarrow f'(x) = -\sin x$$

$$\Rightarrow f''(x) = -\cos x$$

At $x = \pi$, $f''(\pi) = -\cos\pi = 1$

$\therefore$ Our assumption is true.

$\therefore$ At $x = -\pi$

$$f''(-\pi) = -\cos(-\pi) = 1$$

Alternate

Since the function is twice differentiable

$\therefore f''(x) = \text{const. } \forall x f''(-\pi) = -f''(\pi) = 1$

25 (b)

We have,

$$x = \int_0^y \frac{1}{\sqrt{1+4t^2}} dt$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1+4y^2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{1+4y^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{4y}{\sqrt{1+4y^2}}\frac{dy}{dx} = \frac{4y}{\sqrt{1+4y^2}} \times \sqrt{1+4y^2}$$
$$= 4y$$

26 (b)

Given, $3f(x) - 2f\left(\frac{1}{x}\right) = x$ ... (i)

Let $\frac{1}{x} = y$, then $3f\left(\frac{1}{y}\right) - 2f(y) = \frac{1}{y}$

$$\Rightarrow -2f(y) + 3f\left(\frac{1}{y}\right) = \frac{1}{y}$$

$$\Rightarrow -2f(x) + 3f\left(\frac{1}{x}\right) = \frac{1}{x} \text{ ... (ii)}$$

Multiply Eq. (i) by 3 and Eq. (ii) by 2 and then adding, we get

$$5f(x) = 3x + \frac{2}{x}$$

$$\Rightarrow f(x) = \frac{1}{5}\left[3x + \frac{2}{x}\right]$$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{1}{5}\left[3 - \frac{2}{x^2}\right]$$

$$\Rightarrow f'(2) = \frac{1}{5}\left[3 - \frac{2}{4}\right] = \frac{1}{2}$$

27 (d)

Let $y = \log x$

On differentiating w.r.t. x from 1 to n times, we get

$$y_1 = \frac{1}{x}, y_2 = -\frac{1}{x^2}, y_3 = \frac{2}{x^3}, y_4 = -\frac{6}{x^4}, \dots,$$

$\therefore$ by symmctry.

$$y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$$

28 (d)

$$\because x = 2t - |t| = \begin{cases} t, & t \geq 0 \\ 3t, & t < 0 \end{cases}$$

$$\therefore t = \begin{cases} x, & x \geq 0 \\ x/3, & x < 0 \end{cases}$$

$$\therefore y = t^2 + t|t| = \begin{cases} 2t^2, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$= \begin{cases} 2x^2, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Hence, $y'(x) = \begin{cases} 4x, & x > 0 \\ 0, & x < 0 \end{cases}$

$\therefore$ We can't find $\frac{dx}{dt}$ as the derivative does not exist at $t = 0$

29 (c)

We have,

$$f(x) = (x^3 + 2)^{30}$$

Clearly, it is a polynomial of degree 90

It is given that $f^n(x)$ is a polynomial of degree 20.

Therefore,

$\therefore 90 - \pi = 20 \Rightarrow n = 70$

30 (d)

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$$

$$\Rightarrow y = \sqrt{x + y}$$

On squaring both sides, we get

$$y^2 = x + y \Rightarrow 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y - 1) = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y-1}$$

31 (d)

Given, $f(x)$ is a polynomial of degree 3.

$\therefore f(x) = x^3 + ax^2 + bx + c$

But $f(x) = x^3 + x^2 + f'(1) + xf''(2) + f'''(3)$

$\therefore a = f'(1), b = f''(2), c = f'''(3)$

Now, $f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$

$f'''(x) = 6$

$\therefore a = f'(1) = 3 + 2a + b$

$\Rightarrow -a - b = 3$ ...(i)

$b = 12 + 2a$

$\Rightarrow -2a + b = 12$ ...(ii)

and $c = 6$ ...(iii)

On solving Eqs. (i), (ii) and (iii), we get

$a = -5, b = 2$ and $c = 6$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

Thus, $f(0) = 6, f(1) = 4, f(2) = -2, f(3) = -6$

32 (a)

In the given equation put $x = \tan\theta$, we get

$$y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\tan\frac{\theta}{2} = \frac{\theta}{2}$$

$$\Rightarrow y = \frac{1}{2}\tan^{-1}x$$

$$\Rightarrow y' = \frac{1}{2(1+x^2)} \Rightarrow y'(0) = \frac{1}{2}$$

33 (c)

We have,

$y = e^{\sin^{-1}x}$ and $u = \log x$

$$\therefore \frac{dy}{dx} = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}, \frac{du}{dx} = \frac{1}{x} \Rightarrow \frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = \frac{x\, e^{\sin^{-1}x}}{\sqrt{1-x^2}}$$

34 (c)

$$f\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{2}\cos\pi\cos 2\pi\cos 4\pi = 0$$

...(i)

Taking log on given function, we get

$\log f(x) = \log\cos x$

$\qquad\qquad + \log\cos 2x$

$\qquad\qquad + \log\cos 4x$

$\qquad\qquad + \log\cos 8x + \log\cos 16\,x$

$\Rightarrow \dfrac{1}{f(x)} f'(x) = -\tan x$
$\qquad\qquad -2\tan 2x$
$\qquad\qquad -4\tan 4x - 8\tan 8x - 16\tan 16x$
$\Rightarrow f'(x) = -f(x)[\tan x$
$\qquad\qquad +2\tan 2x$
$\qquad\qquad +4\tan 4x$
$\qquad\qquad +8\tan 8x + 16\tan 16x]$
$\Rightarrow f'\left(\dfrac{\pi}{4}\right)$
$\quad = -f\left(\dfrac{\pi}{4}\right)\Big[\tan \dfrac{\pi}{4}$
$\qquad\qquad +2\tan \dfrac{\pi}{2}$
$\qquad\qquad +4\tan \pi + 8\tan 2\pi + 16\tan 4\pi\Big]$
$\quad = 0 \qquad \text{[using eq.(i)]}$

35 (c)

Given, $y = \sqrt{\log x + y} \quad \Rightarrow \quad y^2 = \log x + y$

$\Rightarrow 2y\dfrac{dy}{dx} = \dfrac{1}{x} + \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x(2y-1)}$

36 (d)

Given, $y = \cos^{-1}\cos x$

For $\le x \le \pi, y = x$

$\Rightarrow \dfrac{dy}{dx} = 1$

For $\pi < x \le 2\pi, y = \cos^{-1}\cos(2\pi - x)$

$\Rightarrow y = -x$

$\Rightarrow \dfrac{dy}{dx} = -1$

Hence, option (d) is correct.

37 (b)

Given, $x = a(\cos\theta + \theta\sin\theta)$

and $y = a(\sin\theta - \theta\cos\theta)$

$\therefore \dfrac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta) = a\,\theta\cos\theta$

and $\dfrac{dy}{d\theta} = a(\cos\theta + \theta\sin\theta - \cos\theta) = a\,\theta\sin\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \tan\theta$

38 (d)

Given, $y = \log(\sin(x^2))$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{1}{\sin x^2} . \cos x^2 . 2x = 2x\cot x^2$

At $x = \dfrac{\sqrt{\pi}}{2}, \dfrac{dy}{dx} = \dfrac{2\sqrt{\pi}}{2}\cot\left(\dfrac{\sqrt{\pi}}{2}\right)^2$

$\quad = \sqrt{\pi}\cot\left(\dfrac{\pi}{4}\right) = \sqrt{\pi}$

39 (c)

We have,

$y = a^{x^{a^{x\cdots\infty}}}$

$\Rightarrow y = a^{x^y}$

$\Rightarrow \log y = x^y \log a$

$\Rightarrow \log(\log y) = y\log x + \log(\log a)$

Differentiating w.r.t. x, we get

$\dfrac{1}{\log y}\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{dy}{dx}\cdot\log x + \dfrac{y}{x} \Rightarrow \dfrac{dy}{dx}$

$\qquad\qquad = \dfrac{y^2 \log y}{x(1 - \log x \log y)}$

40 (b)

$f(1) = 5, f'(x) = nx^{n-1}$ so $f'(1) = n$

$f''(1) = n(n-1), \dots f^n(1) = 1.2.\dots.n$

Thus, $f(1) + \dfrac{f'(1)}{1!} + \dots + \dfrac{f^n(1)}{n!}$

$= 5 + \dfrac{n}{1} + \dfrac{n(n-1)}{2!} + \dots + \dfrac{n!}{n!}$

$= (1+1)^n + 4 = 2^n + 4$

41 (a)

Given that, $x = e^t \sin t, y = e^t \cos t \dots$ (i)

At point $(1,1), 1 = e^t \sin t, 1 = e^t \cos t$

$\tan t = 1 \Rightarrow t = \dfrac{\pi}{4}$

On differentiating Eq. (i) w.r.t. x, we get

$\dfrac{dy}{dt} = e^t(\cos t - \sin t)$

and $\dfrac{dx}{dt} = e^t(\sin t + \cos t)$

$\therefore \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\cos t - \sin t}{\cos t + \sin t}$

Again differentiating w.r.t. x, we get

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{\cos t - \sin t}{\cos t - \sin t}\right)\dfrac{dt}{dx}$

$= \left[\dfrac{[(\cos t + \sin t)(-\sin t - \cos t) - (\cos t - \sin t)(-\sin t + \cos t)]}{(\cos t + \sin t)^2}\right]\dfrac{dt}{dx}$

$= \dfrac{-2}{(\cos t + \sin t)^2}\cdot\dfrac{1}{e^t(\sin t + \cos t)}$

$= \dfrac{-2}{(e^t\cos t + e^t\sin t)}\cdot\dfrac{1}{(\cos t + \sin t)^2}$

$= \dfrac{-2}{x+y}\cdot\dfrac{1}{(\cos t + \sin t)^2} \quad \text{[from Eq. (i)]}$

At $t = \dfrac{\pi}{4}, x = 1, y = 1$

$\therefore \dfrac{d^2y}{dx^2} = \dfrac{-2}{1+1}\cdot\dfrac{1}{\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)^2}$

$= \dfrac{-1}{\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]} = -\dfrac{1}{2}$

42 (c)

$e^x + e^y = e^{x+y} = e^x e^y$

$\Rightarrow e^{-y} + e^{-x} = 1$

On differentiating, we get

$$-e^{-y}\frac{dy}{dx} + e^{-x}(-1) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{-x}}{-e^{-y}} \Rightarrow \frac{dy}{dx} = -e^{y-x}$$

43 (c)

$$\frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right)\frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$$

$$= \sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$$

$$\left(\frac{(3-2x)(2)-(2x+3)(-2)}{(3-2x)^2}\right)$$

$$= \frac{12}{(3-2x)^2}\sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{12}{(3-2)^2}\sin\log(5)$$

$$= 12\sin\log 5$$

44 (a)

Let $y = f(\tan x)$ and $u = g(\sec x)$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = f'(\tan x)\sec^2 x$$

and $\frac{du}{dx} = g'(\sec x)\sec x \tan x$

$$\therefore \frac{dy}{du} = \frac{dy/dx}{du/dx} = \frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x \tan x}$$

$$\therefore \left(\frac{dy}{du}\right)_{x=\frac{\pi}{4}} = \frac{f'\left(\tan\frac{\pi}{4}\right)}{g'\left(\sec\frac{\pi}{4}\right)\sin\frac{\pi}{4}}$$

$$= \frac{f'(1)\cdot\sqrt{2}}{g'(\sqrt{2})} = \frac{2\cdot\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$$

45 (a)

$$x^{2x} - 2x^x\cot y - 1 = 0 \quad ...(i)$$

At $x = 1$,

$$1 - 2\cot y - 1 = 0$$

$$\Rightarrow \cot y = 0 \Rightarrow y = \frac{\pi}{2}$$

On differentiating Eq. (i), w.r.t.$'x'$, we get

$$2x^{2x}(1 + \log x) - 2$$

$$\left[x^x(-\text{cosec}^2 y)\frac{dy}{dx} + \cot y\, x^x(1+\log x)\right] = 0$$

At $\left(1, \frac{\pi}{2}\right)$,

$$2(1+\log 1) - 2\left(1(-1)\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} + 0\right) = 0$$

$$\Rightarrow 2 + 2\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = 0 \Rightarrow \left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = -1$$

46 (b)

Given, $\frac{y}{x} = \log x - \log(a+bx)$

$$\Rightarrow \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{1}{a+bx}b = \frac{a}{x(a+bx)}$$

$$\Rightarrow x\frac{dy}{dx} - y = \frac{ax}{a+bx} \quad ...(i)$$

$$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx}$$

$$= \frac{(a+bx)a - ax\cdot b}{(a+bx)^2}$$

$$\Rightarrow x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$$

$$\Rightarrow x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2 \quad [\text{using eq(i)}]$$

47 (a)

We have,

$$y = x^{\sin x} + \sqrt{x}$$

$$\Rightarrow y = e^{\sin x \log x} + \sqrt{x}$$

$$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right) + \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \frac{\pi}{2}\left(0 + \frac{2}{\pi}\right) + \frac{1}{2\sqrt{\frac{\pi}{2}}} = 1 + \frac{1}{\sqrt{2\pi}}$$

48 (c)

We have, $\phi(x) = f^{-1}(x)$

$$\Rightarrow x = f[\phi(x)]$$

On differentiating both sides w.r.t. x, we get

$$1 = f'[\phi(x)]\cdot\phi'(x)$$

$$\Rightarrow \phi(x) = \frac{1}{f'[\phi(x)]} \quad ...(i)$$

Since, $f'(x) = \frac{1}{1+x^5}$ (given)

$$\Rightarrow f'[\phi(x) = \frac{1}{1+[\phi(x)]^5}$$

From Eq.(i),

$$\phi'(x) = \frac{1}{f'[\phi(x)]} = 1 + [\phi(x)]^5$$

49 (c)

$$\frac{d}{dx}\left[\frac{e^x+1}{e^x}\right] = \frac{d}{dx}[1 + e^{-x}] = -e^{-x} = -\frac{1}{e^x}$$

50 (a)

We have, $f(x) = x^3 - 3x^2 + 2x + 10$

$$f'(x) = 3x^2 - 6x + 2$$

Put $f'(x) = 0$, $3x^2 - 6x + 2 = 0$

$$\Rightarrow x = \frac{6 \pm \sqrt{36-24}}{2\times 3}$$

$$= \frac{3+\sqrt{3}}{3} \quad \left[\because x = \frac{3-\sqrt{3}}{3} \text{ does not lie in the given interval}\right]$$

51 (b)

We have,

$$x^2 + y^2 = a^2 \Rightarrow x + yy_1 = 0 \Rightarrow y_1 = -x/y$$

Now, $yy_1 + x = 0$

$$\Rightarrow y\,y_2 + y_1^2 + 1 = 0 \quad [\text{Differentiating w.r.t. } x]$$

$$\Rightarrow y = -\frac{1+y_1^2}{y^2} \quad ... (i)$$

$$\therefore\ k = \frac{1}{a}\left|\frac{1}{\sqrt{x^2+y^2}}\right| = \left\{\frac{1}{y\sqrt{1+\frac{x^2}{y^2}}}\right\}$$

$$\Rightarrow k = \left\{\frac{1}{y\sqrt{1+y_1^2}}\right\} \qquad [\because y_1 = -x/y]$$

$$\Rightarrow k = \left\{\frac{y''}{(1+y_1^2)\sqrt{1+y_1^2}}\right\} = \frac{|y''|}{(1+y_1^2)^{3/2}}$$

52 (c)

Let $y = \frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x \tan^2 x}$

$$= \frac{(\tan 2x - \tan x)}{(1 + \tan 2x \tan x)} \cdot \frac{(\tan 2x + \tan x)}{(1 - \tan 2x \tan x)}$$

$$= \tan(2x - x)\tan(2x + x)$$

$$= \tan x \tan 3x$$

On differentiating w.r.t. x, we get

$$\frac{d}{dx}(y \cdot \cot 3x) = \frac{d}{dx}(\tan x \tan 3x \cot 3x)$$

$$= \frac{d}{dx}(\tan x)$$

$$= \sec^2 x$$

53 (b)

Given, $\frac{x}{1} = \frac{1 - \sqrt{y}}{1 + \sqrt{y}}$

Applying componendo and dividendo, we get

$$\frac{1 + x}{1 - x} = \frac{(1 + \sqrt{y}) + (1 - \sqrt{y})}{(1 + \sqrt{y}) - (1 - \sqrt{y})}$$

$$\Rightarrow \frac{1 + x}{1 - x} = \frac{2}{2\sqrt{y}}$$

$$\Rightarrow y = \left(\frac{1 - x}{1 + x}\right)^2$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{-2(1 + x)^2(1 - x) - (1 - x)^2 \cdot 2(1 + x)}{(1 + x)^4}$$

$$= \frac{(1 - x)(1 + x)(-2 - 2x - 2 + 2x)}{(1 + x)^4}$$

$$= \frac{4(x - 1)}{(x + 1)^3}$$

54 (b)

We have,

$$y = \cos^{-1}\left(\frac{2\cos x - 3\sin x}{\sqrt{13}}\right)$$

$$\Rightarrow y = \cos^{-1}\left\{\frac{2}{\sqrt{13}}\cos x - \sqrt{1 - \frac{4}{13}}\sqrt{1 - \cos^2 x}\right\}$$

$$\Rightarrow y = \cos^{-1}(\cos x) = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$$

$$\Rightarrow y = x - \cos^{-1}\left(\frac{2}{\sqrt{13}}\right) \Rightarrow \frac{dy}{dx} = 1$$

55 (d)

Given, $y = \sin(\log_e x)$ ...(i)

$$\Rightarrow \frac{dy}{dx} = \cos(\log_e x) \cdot \frac{1}{x} \quad ...(ii)$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{-x \cdot \sin(\log_e x) \cdot \frac{1}{x} = \cos(\log_e x) \cdot 1}{x^2}$$

$$= \frac{-\sin(\log_e x) - \cos(\log_e x)}{x^2}$$

$$x^2 \frac{d^2 y}{dx^2} = -\sin(\log_e x) - x\frac{dy}{dx} \quad \text{[using Eq. (ii)]}$$

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} = -y \quad \text{[using Eq. (i)]}$$

56 (c)

Since, $y = \sin^{-1}\frac{x}{2} + \cos^{-1}\frac{x}{2} = \frac{\pi}{2}$

$$\therefore \frac{dy}{dx} = 0$$

57 (d)

$$\because \quad y = \cos(\sin x^2)$$

$$\therefore \quad \frac{dy}{dx} = -\sin(\sin x^2)\cos x^2 \cdot 2x$$

At $x = \sqrt{\frac{\pi}{2}}, \frac{dy}{dx} = -\sin\left(\sin\frac{\pi}{2}\right)\cos\frac{\pi}{2} \cdot 2 \cdot \sqrt{\frac{\pi}{2}}$

$$= 0 \quad \left[\because \cos\frac{\pi}{2} = 0\right]$$

58 (b)

Let $f(x) = \cos x, f'(x) = \sin x$

$$\Rightarrow f''(x) = -\cos x$$

$$\Rightarrow f'''(x) = \sin x$$

Since, $\sin x$ is an odd function.

$\therefore f'''$ is an odd function.

59 (b)

Given, $\sec z = \frac{x^4 + y^4 - 8x^2 y^2}{x^2 + y^2}$

Here, $n = 2$

$$\therefore x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2 \cdot \frac{\sec z}{\sec z \cdot \tan z} = 2\cot z$$

60 (c)

We have,

$2^x + 2^y = 2^{x+y}$

$$\Rightarrow 2^x \log 2 + 2^y \log 2\frac{dy}{dx} = 2^{x+y} \log 2\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow (2^y - 2^{x+y})\frac{dy}{dx} = 2^{x+y} - 2^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^x(2^y - 1)}{2^y(1 - 2^x)} = 2^{x-y}\left(\frac{2^y - 1}{1 - 2^x}\right)$$

61 (b)

We have,

$f(x) = 10\cos x + (13 + 2x)\sin x$

$$\Rightarrow f'(x) = -10\sin x + 2\sin x + (13 + 2x)\cos x$$

$$\Rightarrow f''(x) = -8\cos x - (13 + 2x)\sin x + 2\cos x$$

$$\therefore f''(x) + f(x) = 4\cos x$$

62 (b)

$xy = \tan^{-1}(xy) + \cot^{-1}(xy) = \dfrac{\pi}{2}$

$\Rightarrow xy = \dfrac{\pi}{2}$

On differentiating, we get

$\therefore\ x\dfrac{dy}{dx} + y = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-y}{x}$

63 (a)

Since, $x^y \cdot y^x = 16$

$\therefore \log_e x^y + \log_e y^x = \log_e 16$

$\Rightarrow y\log_e x + x\log_e y = 4\log_e 2$

Now, on differentiating both sides w.r.t. x, we get

$\dfrac{y}{x} + \log_e x\dfrac{dy}{dx} + \dfrac{x}{y}\dfrac{dy}{dx} + \log_e y\cdot 1 = 0$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{\left(\log_e y + \dfrac{y}{x}\right)}{\left(\log_e x + \dfrac{x}{y}\right)}$

$\therefore \dfrac{dy}{dx}\bigg|_{(2,2)} = -\dfrac{(\log_e 2 + 1)}{(\log_e 2 + 1)} = -1$

64 (c)

We have,

$y = b\sin\theta,\ x = a\cos\theta$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{b}{a}\cot\theta$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{b}{a}\operatorname{cosec}^2\theta\dfrac{d\theta}{dx} = -\dfrac{b}{a^2}\operatorname{cosec}^3\theta$

$\Rightarrow \dfrac{d^3y}{dx^3} = \dfrac{b}{a^2}3\operatorname{cosec}^2\theta(-\operatorname{cosec}\theta + \cot\theta)\dfrac{d\theta}{dx}$

$\Rightarrow \dfrac{d^3y}{dx^3} = \dfrac{3b}{a^2}\operatorname{cosec}^3\theta\cot\theta \times \dfrac{-1}{a\sin\theta}$

$= -\dfrac{3\,b}{a^3}\operatorname{cosec}^4\theta\cot\theta$

65 (b)

Given, $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\psi'}{\phi'}$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{\psi'}{\phi'}\right)\dfrac{dt}{dx}$

$= \dfrac{\phi'\psi'' - \psi'\phi''}{(\phi')^2}\cdot\dfrac{1}{\phi'}$

66 (b)

Since, u is not a homogeneous function. But

$f(x,y) = \sin u = \dfrac{x^2+y^2}{x+y}$ is a homogeneous function

of degree one.

Here, by Euler's theorem,

$x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = f$

$\Rightarrow x\dfrac{\partial}{\partial x}(\sin u) + y\dfrac{\partial}{\partial y}(\sin u) = \sin u$

$\Rightarrow x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$

67 (c)

We have,

$y = \sec^{-1}\left(\dfrac{x+1}{x-1}\right) + \sin^{-1}\left(\dfrac{x-1}{x+1}\right)$

$\Rightarrow y = \cos^{-1}\left(\left(\dfrac{x+1}{x-1}\right)\right) + \sin^{-1}\left(\dfrac{x-1}{x+1}\right) = \dfrac{\pi}{2}$

$\therefore \dfrac{dy}{dx} = 0$

68 (a)

Given, $y = x^n\log x + x(\log x)^n$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = nx^{n-1}\log x + x^n\cdot\dfrac{1}{x} + xn(\log x)^{n-1}\left(\dfrac{1}{x}\right)$

$\qquad\qquad + 1\cdot(\log x)^n$

$= x^{n-1}(1 + n\log x) + (\log x)^{n-1}[n + \log x]$

69 (d)

We have,

$y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \cdots \infty}}}}$

$\Rightarrow y^2 = x + \sqrt{y + \sqrt{x + \sqrt{y + \cdots \infty}}}$

$\Rightarrow y^2 = x + \sqrt{y + y} \Rightarrow (y^2 - x)^2 = 2y$

Differentiating both sides w.r.t. x, we get

$2(y^2 - x)\left(2y\dfrac{dy}{dx} - 1\right) = 2\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx}$

$\qquad\qquad = \dfrac{y^2 - x}{y^3 - xy - 1}$

70 (c)

We have,

$\dfrac{d}{dt}\left(\int_a^b f(x,t)\,dx\right)$

$\qquad = \int_a^b \dfrac{\partial f}{\partial t}\,dx + \dfrac{\partial b}{\partial t}f(b,t)$

$\qquad\qquad - \dfrac{\partial a}{\partial t}f(a,t)$

$\therefore y = \int_0^x f(t)\sin\{k(x-t)\}\,dt$

$\Rightarrow \dfrac{dy}{dx} = \int_0^x kf(t)\cos\{k(x-t)\}\,dt$

Differentiating again w.r.t. x, we get

$\dfrac{d^2y}{dx^2} = \int_0^x -k^2 f(t)\sin\{k(x-t)\}\,dt$

$\qquad\qquad + kf(x)\{\cos\{k(x-x)\} - 0$

$$\Rightarrow \frac{d^2y}{dx^2} = -k^2 y + kf(x) \Rightarrow \frac{d^2y}{dx^2} + k^2 y = kf(x)$$

71 (b)

Given,

$$f(x) = 10 \cos x + (13 + 2x) \sin x$$
$$\Rightarrow f'(x) = -10 \sin x + (13 + 2x) \cos x + 2 \sin x$$
$$\Rightarrow f''(x) = -10 \cos x$$
$$\qquad\qquad - (13 + 2x) \sin x$$
$$\qquad\qquad + 2 \cos x + 2 \cos x$$
$$\Rightarrow f''(x) = -f(x) + 4 \cos x$$
$$\therefore f''(x) + f(x) = 4 \cos x$$

72 (a)

If $f(x) = ax^{41} + bx^{-40}$, then
$$f'(x) = 41ax^{40} - 40bx^{-41}$$
$$\Rightarrow f''(x) = 1640 \, ax^{39} + 1640 \, bx^{-42}$$
$$\Rightarrow f''(x) = \frac{1640}{x^2}(ax^{41} + bx^{-40})$$
$$\Rightarrow f''(x) = \frac{1640}{x^2} f(x) \Rightarrow \frac{f''(x)}{f(x)} = 1640x^{-2}$$

So, statement-1 is true

We have,

$$\tan^{-1}\frac{2x}{1-x^2} = \begin{cases} \pi + 2\tan^{-1}x & x < -1 \\ 2\tan^{-1}x, \text{ if } & -1 \, x \le 1 \\ -\pi + 2\tan^{-1}x & x > 1 \end{cases}$$

$$\therefore \frac{d}{dx}\left(\tan^{-1}\frac{2x}{1-x^2}\right) = \frac{2}{1+x^2} \text{ for all } x$$

So, statement-II is not true

73 (a)

$$\frac{d}{dx}\left[\cos x^0\right] = \frac{d}{dx}\left[\cos\frac{\pi x}{180}\right]$$

$$\begin{bmatrix} \because 1^0 = \frac{\pi}{180} \text{ radians} \\ \therefore x^0 = \frac{\pi x}{180} \text{ radians} \end{bmatrix}$$

$$= -\sin\left(\frac{\pi x}{180}\right) \cdot \frac{\pi}{180} = -\frac{\pi}{180}\sin(x^0)$$

74 (a)

We have,

$$y = e^{1+\log x} \Rightarrow y = e \, e^{\log_e x} = e^x \Rightarrow \frac{dy}{dx} = e$$

75 (a)

Given, $y = \cos x + x \cos y = \pi$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}\cos x + y(-\sin x) + x(-\sin y)\frac{dy}{dx} + \cos y = 0$$
$$\Rightarrow \frac{dy}{dx} = \frac{y \sin x - \cos y}{\cos x - x \sin y}$$

Again differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} =$$

$$\frac{\left[\begin{array}{c} (\cos x - x\sin y)\left(y\cos x + \sin x\frac{dy}{dx} + \sin y\frac{dy}{dx}\right) \\ -(y\sin x - \cos y)\left(-\sin x - \sin y - x\cos y\frac{dy}{dx}\right) \end{array}\right]}{(\cos x - x\sin y)^2}$$

At $x = 0$,

$$f''(0) = \frac{1(y + \sin y) - (-1)(-\sin y)}{(1-0)^2} = y$$

As $y \cos x + x \cos y = \pi$

$\therefore$ At $x = 0 \Rightarrow y = \pi$

Hence, $f''(0) = \pi$

76 (d)

Given that, $x = a \sin \theta$ and $y = b \cos \theta$

On differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = a \cos \theta \text{ and } \frac{dy}{d\theta} = -b \sin \theta$$
$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b}{a}\tan \theta$$

Again differentiating w.r.t. θ, we get

$$\frac{d^2y}{dx^2} = \frac{-b}{a}\sec^2\theta\frac{d\theta}{dx}$$
$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b}{a}\sec^2\theta\frac{1}{a\cos\theta} = -\frac{b}{a^2}\sec^3\theta$$

77 (d)

Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and $v = \cos^{-1}\sqrt{1-x^2}$

On differentiating w.r.t. x, respectively, we get

$$\frac{du}{dx} = \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2} \cdot \left[\frac{(1-x^2)2 - 2x(-2x)}{(1-x^2)^2}\right]$$
$$= \frac{2 + 2x^2}{(1+x^2)^2} = \frac{2}{1+x^2}$$

and $\frac{dv}{dx} = -\frac{1}{\sqrt{1-(1-x^2)}} \cdot \left[\frac{(-2x)}{2\sqrt{1-x^2}}\right]$

$$= \frac{1}{\sqrt{x^2}}\left[\frac{x}{\sqrt{1-x^2}}\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{du}{dv} = \frac{\frac{2}{1+x^2}}{\frac{1}{\sqrt{1-x^2}}} = \frac{2\sqrt{1-x^2}}{1+x^2}$$

Alternate

Let $\quad u = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$$\Rightarrow \quad \frac{du}{dx} = \frac{2}{1+x^2}$$

and let $v = \cos^{-1}\sqrt{1-x^2} = \sin^{-1}x$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Now, $\frac{du}{dv} = \frac{2}{1+x^2}\bigg/\frac{1}{\sqrt{1-x^2}} = \frac{2\sqrt{1-x^2}}{1+x^2}$

78 (c)

We have,

$$y = \sin^{-1}\{\sqrt{x - ax} - \sqrt{a - ax}\}$$
$$\Rightarrow y = \sin^{-1}\left\{\sqrt{x}\sqrt{1-(\sqrt{a})^2} - \sqrt{a}\sqrt{1-(\sqrt{x})^2}\right\}$$
$$\Rightarrow y = \sin^{-1}\sqrt{x} - \sin^{-1}\sqrt{a}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{1}{2\sqrt{x(1-x)}}$$

79 (b)

Let $p = \sin^{-1} \dfrac{2x}{1+x^2} = 2 \tan^{-1} x$

and $q = \cos^{-1} \dfrac{1-x^2}{1+x^2} = 2 \tan^{-1} x$

$\Rightarrow \dfrac{dp}{dx} = \dfrac{2}{1+x^2}, \dfrac{dq}{dx} = \dfrac{2}{1+x^2}$

$\therefore \dfrac{dp}{dq} = \dfrac{\frac{dp}{dx}}{\frac{dq}{dx}} = \dfrac{\frac{2}{(1+x^2)}}{\frac{2}{(1+x^2)}} = 1$

80 **(b)**

$f(x) = e^x$ and $g(x) = \sin^{-1} x$

and $h(x) = f[g(x)]$

$\Rightarrow h(x) = f(\sin^{-1} x) = e^{\sin^{-1} x}$

$\therefore h(x) = e^{\sin^{-1} x}$

$\Rightarrow h'(x) = e^{\sin^{-1} x} \dfrac{1}{\sqrt{1-x^2}}$

$\Rightarrow \dfrac{h'(x)}{h(x)} = \dfrac{1}{\sqrt{1-x^2}}$

81 **(a)**

Given, $x = a(\theta - \sin \theta)$ and $y = a(1 - \cos \theta)$

$\dfrac{dx}{d\theta} = a(1 - \cos \theta)$ and $\dfrac{dy}{d\theta} = a \sin \theta$

$\therefore \dfrac{dy}{dx} = \dfrac{a \sin \theta}{a(1 - \cos \theta)}$

$= \dfrac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = \cot \dfrac{\theta}{2}$

82 **(c)**

We have,

$x = a \cos \theta + \dfrac{1}{2} b \cos 2\theta$, $y = a \sin \theta + \dfrac{1}{2} b \sin 2\theta$

$\therefore \dfrac{dx}{d\theta} = -a \sin \theta - b \sin 2\theta, \dfrac{dy}{d\theta}$

$\qquad\qquad = a \cos \theta + b \sin 2\theta$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{a \cos \theta + b \cos 2\theta}{-a \sin \theta - b \sin 2\theta}$

$\Rightarrow \dfrac{d^2 y}{dx^2} = -\dfrac{d}{d\theta}\left(\dfrac{a \cos \theta + b \cos 2\theta}{a \sin \theta + b \sin 2\theta}\right)\dfrac{d\theta}{dx}$

$\therefore \dfrac{d^2 y}{dx^2} = 0$

$\Rightarrow (a \sin \theta + b \sin 2\theta)(-a \sin \theta - b \sin 2\theta)$

$= (a \cos \theta + b \cos 2\theta)(a \cos \theta + 2 b \cos 2\theta)$

$\Rightarrow -a^2 - 2 b^2 - 3 ab(\cos 2\theta \cos \theta + \sin \theta \sin 2\theta)$

$\qquad\qquad = 0$

$\Rightarrow a^2 + 2 b^2 = 3 a b \cos \theta \Rightarrow \cos \theta = \dfrac{a^2 + 2 b^2}{3 ab}$

83 **(d)**

$\because y = \log_a x + \dfrac{\log a}{\log x} + 1 + 1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x}\log_a e - \log a \left(\dfrac{1}{\log x}\right)^2 \dfrac{1}{x}$

$= \dfrac{1}{x \log a} - \dfrac{\log a}{x(\log x)^2}$

84 **(d)**

Given, $y = \sin[\cos^{-1}\{\sin(\cos^{-1} x)\}]$

$\Rightarrow y = \sin\left[\cos^{-1}\left\{\sin\left(\dfrac{\pi}{2} - \sin^{-1} x\right)\right\}\right]$

$= \sin[\cos^{-1}(\cos \sin^{-1} x)]$

$= \sin(\sin^{-1} x)$

$\Rightarrow \qquad y = x$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = 1$

85 **(c)**

We have, $8f(x) + 6f\left(\dfrac{1}{x}\right) = x+5$ for all x ...(i)

Therefore, $8f\left(\dfrac{1}{x}\right) + 6f(x) = \dfrac{1}{x}+5$ ...(ii)

From Eqs. (i) and (ii), we have

$f(x) = \dfrac{1}{28}\left(8x - \dfrac{6}{x} + 10\right)$

Now, $y = x^2 f(x)$

$\Rightarrow \qquad y = \dfrac{1}{28}(8x^2 - 6x + 10x^2)$

$\Rightarrow \qquad \dfrac{dy}{dx} = \dfrac{1}{28}(24x^2 + 20x - 6)$

$\therefore \left(\dfrac{dy}{dx}\right)_{x=-1} = \dfrac{1}{28}(24 - 20 - 6)$

$= -\dfrac{1}{14}$

86 **(d)**

Given, $x = \sin t$, $y = \cos pt$

$\dfrac{dx}{dt} = \cos t, \quad \dfrac{dy}{dt} = -p \sin pt$

$\therefore \quad \dfrac{dy}{dx} = -\dfrac{p \sin pt}{\cos t}$

$\Rightarrow y_1 = \dfrac{-p\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\Rightarrow y_1 \sqrt{1-x^2} = -p\sqrt{1-y^2}$

$\Rightarrow y_1^2(1-x^2) = p^2(1-y^2)$

$\Rightarrow 2y_1 y_2(1-x^2) - 2xy_1^2 =$

$-2yy_1 p^2$ [differentiate]

$\Rightarrow (1 - x^2)y_2 - xy_1 + p^2 y = 0$

87 **(b)**

On taking log in the given equation, we get

$\log y = \log(1 - x) + \log(2 - x) + \ldots + \log(n - x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{(1-x)}(-1) + \dfrac{1}{(2-x)}(-1) + \ldots + \dfrac{1}{(n-x)}(-1)$

$\dfrac{dy}{dx} = y$

$\left[\dfrac{\begin{array}{l}(2-x)(3-x)\ldots(n-x)+\\(1-x)(3-x)\ldots(n-x)+\ldots\end{array}}{y}\right](-1)$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{x=1} = 1.2 \ldots (n-1)(-1)$

$= (-1)(n-1)!$

88 (c)

$\because f(x) = \sqrt{ax} + \dfrac{a^2}{\sqrt{ax}}$

On differentiating w.r.t. x, we get

$f'(x) = \dfrac{\sqrt{a}}{2\sqrt{x}} + \dfrac{a^2}{\sqrt{a}}\left(\dfrac{-1}{2}x^{-3/2}\right)$

$\Rightarrow f'(x) = \dfrac{\sqrt{a}}{2\sqrt{x}} - \dfrac{a^2}{2\sqrt{a}}x^{-3/2}$

$\therefore f'(a) = \dfrac{\sqrt{a}}{2\sqrt{a}} - \dfrac{a^2}{2\sqrt{a}.\,a^{3/2}}$

$\Rightarrow f'(a) = \dfrac{1}{2} - \dfrac{a^2}{2a^2} = 0$

89 (d)

Given, $y = \log_2 \log_2(x)$

$= \dfrac{\log_e \log_2(x)}{\log_e 2} = \dfrac{\log_e \left[\dfrac{\log_e x}{\log_e 2}\right]}{\log_e 2}$

$\Rightarrow y = \dfrac{\log_e \log_e x - \log_e \log_e 2}{\log_e 2}$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{1}{\log_e 2}\left[\dfrac{1}{x \log_e x} - 0\right] = \dfrac{\log_2 e}{x \log_e x}$

90 (b)

Let $y = f(\sin x)$

$\Rightarrow y = \log(\sin x)$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\sin x}\cos x = \cot x$

91 (c)

Given, $y = e^{ax}\sin bx$ ...(i)

$\Rightarrow \dfrac{dy}{dx} = ae^{ax}\sin bx + be^{ax}\cos bx$

$\Rightarrow \dfrac{dy}{dx} = ay + be^{ax}\cos bx$ ...(ii)

$\Rightarrow \dfrac{d^2y}{dx^2} = a\dfrac{dy}{dx} + abe^{ax}\cos bx - e^{ax}b^2\sin bx$

$\Rightarrow \dfrac{d^2y}{dx^2} = a\dfrac{dy}{dx} + a\left(\dfrac{dy}{dx} - ay\right) - b^2y$

[from Eqs. (i) and (ii)]

$\Rightarrow \dfrac{d^2y}{dx^2} - 2a\dfrac{dy}{dx} + a^2y = -b^2y$

93 (a)

Let $f(x) = ax^2 + bx + c$ ....(i)

$\Rightarrow f'(x) = 2ax + b$

and $f''(x) = 2a$

Given, $f(0) = 4, f'(0) = 3$ and $f''(0) = 4$, we get

$c = 4, \quad b = 3, \quad a = 2$

$\therefore f(x) = 2x^2 + 3x + 4$

$\therefore f(-1) = 2(-1)^2 + 3(-1) + 4 = 3$

94 (a)

We have,

$x = \exp\left\{\tan^{-1}\left(\dfrac{y - x^2}{x^2}\right)\right\}$

$\Rightarrow \log_e x = \tan^{-1}\left(\dfrac{y - x^2}{x^2}\right)$

$\Rightarrow \tan(\log_e x) = \dfrac{y}{x^2} - 1$

$\Rightarrow y = x^2\{1 + \tan(\log_e x)\}$

$\Rightarrow \dfrac{dy}{dx} = 2x\{1 + \tan(\log_e x)\} + x\sec^2(\log_e x)$

95 (a)

We have,

$y^2 = ax^2 + bx + c$

Differentiating this with respect to x, we get

$2y\dfrac{dy}{dx} = 2ax + b$ ...(i)

$\Rightarrow 2\left(\dfrac{dy}{dx}\right)^2 + 2y\dfrac{d^2y}{dx^2} = 2a$

$\Rightarrow y\dfrac{d^2y}{dx^2} = a - \left(\dfrac{dy}{dx}\right)^2$

$\Rightarrow y\dfrac{d^2y}{dx^2} = a - \left(\dfrac{2ax+b}{2y}\right)^2$ ...[From (i)]

$\Rightarrow y\dfrac{d^2y}{dx^2} = \dfrac{4ay^2 - (2ax + b)^2}{4y^2}$

$\Rightarrow 4y^3\dfrac{d^2y}{dx^2} = 4a(ax^2 + bx + c)$
$\qquad\qquad - (4a^2x^2 + 4abx + b^2)$

$\Rightarrow 4y^3\dfrac{d^2y}{dx^2} = 4ac - b^2$

$\Rightarrow y^3\dfrac{d^2y}{dx^2} = \dfrac{4ac - b^2}{4} = $ a const.

96 (b)

Let $u = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = 2\tan^{-1}x$

$\therefore \dfrac{du}{dx} = \dfrac{2}{1+x^2}$

and $v = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$\therefore \dfrac{dv}{dx} = \dfrac{2}{1+x^2}$

Hence, $\dfrac{du}{dv} = \dfrac{\left(\dfrac{du}{dx}\right)}{\left(\dfrac{dv}{dx}\right)} = 1$

97 (d)

$I_n = \dfrac{d^{n-1}}{dx^{n-1}}[x^{n-1} + nx^{n-1}\log x]$

$I_n = (n-1)\dfrac{d^{n-2}}{dx^{n-2}}x^{n-2} + n\dfrac{d^{n-1}}{dx^{n-1}}(x^{n-1}\log x)$

$I_n = (n-1)! + nI_{n-1}$

$\Rightarrow I_n - nI_{n-1} = (n-1)!$

98 (d)

We have,

$y = \sin(\log_e x)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x}\cos(\log_e x)$

$$\Rightarrow x\frac{dy}{dx} = \cos(\log_e x)$$
$$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-\sin(\log_e x)}{x} \Rightarrow x^2\frac{dy}{dx^2} + x\frac{dy}{dx}$$
$$= -y$$

99 (c)

Given, $y - 1 = t^{10}$, $x - 1 = t^8$

$$\Rightarrow \frac{dy}{dt} = 10t^9 \text{ and } \frac{dx}{dt} = 8t^7$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{5}{4}t^2$$

$$= \frac{5}{4}\frac{t^{10}}{t^8} = \frac{5(y-1)}{4(x-1)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{5}{4}\frac{(x-1)\frac{dy}{dx} - (y-1)1}{(x-1)^2}$$

$$= \frac{5}{4}\frac{1}{(x-1)}\left[\frac{dy}{dx} - \frac{(y-1)}{(x-1)}\right]$$

$$= \frac{5}{4}\frac{1}{(x-1)}\left[\frac{5}{4}\cdot\frac{(y-1)}{(x-1)} - \frac{(y-1)}{(x-1)}\right]$$

$$= \frac{5}{4}\frac{(y-1)}{(x-1)^2}\left(\frac{5}{4} - 1\right) = \frac{5}{16t^6}$$

100 (a)

Let $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ and $z = \sin^{-1}(3x - 4x^3)$

$$\Rightarrow y = 2\sin^{-1}x \text{ and } z = 3\sin^{-1}x$$

$$\Rightarrow y = \frac{2}{3}z \quad \therefore \frac{dy}{dz} = \frac{2}{3}$$

101 (c)

Since, $y = (1+x)(1+x^2)(1+x^4)\ldots(1+x^{2^n})$

$$\Rightarrow (1-x)y = (1-x^2)(1+x^2)(1+x^4)\ldots(1+x^{2^n})$$

$$= (1-x^4)(1+x^4)\ldots(1+x^{2^n})$$

$$\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$

$$= (1-x^{2^n})(1+x^{2^n}) = 1 - x^{2^{n+1}}$$

$$\therefore y = \frac{1-x^{2^{n+1}}}{(1-x)}$$

$$\frac{dy}{dx}$$

$$= \frac{(1-x)(-2^{n+1})\cdot x^{2^{n+1}-1} - \left(1-x^{2^{n+1}}\right)(-1)}{(1-x)^2}$$

$$\therefore \frac{dy}{dx}\bigg|_{x=0} = \frac{(1-0)(-2^{n+1}\cdot 0) - (1-0)(-1)}{1} = 1$$

102 (b)

We have,

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

Putting $x = \sin A$, $y = \sin B$, it reduces to

$$\Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}(a)$$

Differentiating w.r.t. x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

103 (c)

Let $y = e^{x^3}$, $z = \log x$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = e^{x^3}(3x^2) = 3x^2e^{x^3} \text{ and } \frac{dz}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{3x^2e^{x^3}}{\left(\frac{1}{x}\right)} = 3x^3e^{x^3}$$

104 (c)

Let $y = \sqrt{x^2 + 16}$ and $z = \frac{x}{x-1}$

On differentiating w.r.t., x, we get

$$\frac{dy}{dx} = \frac{1}{2}(x^2 + 16)^{-1/2}(2x)$$

and $\frac{dz}{dx} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$

$$\therefore \frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{-x}{\sqrt{x^2+16}}\cdot\frac{1}{\frac{1}{(x-1)^2}}$$

$$\left(\frac{dy}{dz}\right)_{x=3} = \frac{-3(2)^2}{\sqrt{25}} = \frac{-12}{5}$$

105 (c)

Given, $x = \log(1+t^2)$ and $y = t - \tan^{-1}t$

$$\frac{dx}{dt} = \frac{1}{1+t^2}\cdot 2t$$

and $\frac{dy}{dt} = 1 - \frac{1}{1+t^2} = \frac{t^2}{1+t^2}$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t^2/(1+t^2)}{2t/(1+t^2)} = \frac{t}{2} \quad \ldots(i)$$

Also, $x = \log(1+t^2) \Rightarrow t^2 = e^x - 1 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$\frac{dy}{dx} = \frac{\sqrt{e^x-1}}{2}$$

106 (c)

$$y$$
$$= \frac{(1-x)(1+x)(1+x^2)(1+x^4)\ldots\ldots(1+x^{2n})}{(1-x)}$$

$$= \frac{1-x^{4n}}{1-x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1-x)(-4nx^{4n-1}) - (1-x^{4n})(-1)}{(1-x)^2}$$

$$= \frac{-4n(1-x)x^{4n-1} + (1-x^{4n})}{(1-x)^2}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = 1$$

107 (b)

We have,

$$f(x) = (1-x)^n$$

$$\therefore f'(x) = -n(1-x)^{n-1}, f''(x) = n(n-1)x^{n-2},$$

$$f'''(x) = -n(n-1)(n-2)x^{n-3} \text{ and so on}$$

$\Rightarrow f(0) = 1, f'(0) = -n, f''(0) = n(n-1),$
$f'''(0) = -n(n-1)(n-2)$ and so on

$\therefore f(0) + f'(0) + \dfrac{f''(0)}{2!} + \cdots + \dfrac{f^n(0)}{n!}$

$= 1 - n + \dfrac{n(n-1)}{n!} + \cdots + (-1)^n \dfrac{n(n-1)\ldots 3.2.1}{n!}$

$= {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \cdots + (-1)^n \; {}^nC_n$

$= (1-1)^n = 0$

108 (a)

$f(x) = (\log_{\cot x} \tan x)(\log_{\tan x} \cot x)^{-1}$
$\qquad\qquad + \tan^{-1} \dfrac{4x}{4-x^2}$

$= \dfrac{\log \tan x}{\log \cot x} \cdot \dfrac{\log \tan x}{\log \cot x} + \tan^{-1}\left(\dfrac{4x}{4-x^2}\right)$

$= \dfrac{(\log \tan x)^2}{(-\log \tan x)^2} + \tan^{-1}\left(\dfrac{4x}{4-x^2}\right)$

$= 1 + \tan^{-1}\left(\dfrac{4x}{4-x^2}\right)$

$\therefore f'(x) = \dfrac{1}{1 + \left(\dfrac{4x}{4-x^2}\right)^2} \cdot \dfrac{(4-x^2)4 - 4x(-2x)}{(4-x^2)^2}$

$= \dfrac{16 - 4x^2 + 8x^2}{(4-x)^2 + 16x^2} = \dfrac{4(4+x^2)}{(4-x^2)^2 + (4x)^2}$

Hence, $f(2) = \dfrac{4(4+4)}{0+(8)^2} = \dfrac{32}{64} = \dfrac{1}{2}$

109 (c)

Given, $y = \cos^2 \dfrac{3x}{2} - \sin^2 \dfrac{3x}{2} = 2\cos^2 \dfrac{3x}{2} - 1$

$\Rightarrow \dfrac{dy}{dx} = 2.2\cos \dfrac{3x}{2}\left(-\sin \dfrac{3x}{2}\right)\left(\dfrac{3}{2}\right)$

$\Rightarrow \dfrac{dy}{dx} = -6\cos \dfrac{3x}{2}\sin \dfrac{3x}{2}$

$\Rightarrow \dfrac{d^2y}{dx^2} = -6\left[\cos \dfrac{3x}{2}\left(\cos \dfrac{3x}{2}\right)\cdot\dfrac{3}{2}\right.$
$\qquad\qquad \left. - \sin \dfrac{3x}{2}\sin \dfrac{3x}{2}\cdot\dfrac{3}{2}\right]$

$\Rightarrow \dfrac{d^2y}{dx^2} = -9\left[\cos^2 \dfrac{3x}{2} - \sin^2 \dfrac{3x}{2}\right] = -9y$

Alternate

$y = \cos^2 \dfrac{3x}{2} - \sin^2 \dfrac{3x}{2}$

$\Rightarrow y = \cos 3x$

$\Rightarrow \dfrac{dy}{dx} = -3\sin 3x$

$\Rightarrow \dfrac{d^2y}{dx^2} = -9\cos 3x = -9y$

110 (d)

Given,

$f(x) = \log_x(\log_e x) = \dfrac{\log_e \log_e x}{\log_e x}$

On differentiating w.r.t. x, we get

$f'(x) = \dfrac{\log_e x.\dfrac{1}{\log_e x}.\dfrac{1}{x} - \log_e \log_e x.\dfrac{1}{x}}{(\log_e x)^2}$

$\Rightarrow f'(x) = \dfrac{1 - \log_e \log_e x}{x(\log_e x)^2}$

$\Rightarrow f'(e) = \dfrac{1 - \log_e \log_e e}{e(\log_e e)^2} = \dfrac{1 - \log_e 1}{e} = \dfrac{1}{e}$

111 (c)

We have,

$f(x) = \cos^{-1}\left\{\dfrac{1 + (\log_e x)^2}{1 + (\log_e x)^2}\right\}$

$\Rightarrow f(x) = 2\tan^{-1}(\log_e x) \quad [\because \log_e x$
$\qquad\qquad > 0$ in the nbd of $x = e]$

$\Rightarrow f'(x) = \dfrac{2}{1 + (\log_e x)^2} \times \dfrac{1}{x} \Rightarrow f'(e) = \dfrac{1}{e}$

112 (c)

Given, $f(x) = 1 + nx + \dfrac{n(n-1)}{2!}x^2$
$\qquad + \dfrac{n(n-1)(n-2)}{3!}x^3 + \cdots + x^n$

$\Rightarrow f(x) = (1+x)^n$

$\Rightarrow f'(x) = n(1+x)^{n-1}$

$\Rightarrow f''(x) = n(n-1)(1+x)^{n-2}$

$\Rightarrow f''(1) = n(n-1)2^{n-2}$

113 (c)

On differentiating w.r.t. x, we get

$2^x \log 2 + 2^y \log 2 \dfrac{dy}{dx} = 2^{x+y}\log 2\left(1 + \dfrac{dy}{dx}\right)$

$\Rightarrow 2^x + 2^y \dfrac{dy}{dx} = 2^{x+y}\left(1 + \dfrac{dy}{dx}\right)$

$\Rightarrow 2^x - 2^{x+y} = \dfrac{dy}{dx}(2^{x+y} - 2^y)$

$\Rightarrow 2^{x-y}\dfrac{(1 - 2^y)}{(2^x - 1)} = \dfrac{dy}{dx}$

114 (a)

We have,

$2x^2 - 3xy + y^2 + x + 2y - 8 = 0$

Differentiating w.r.t. to x, we get

$4x - 3\left(x\dfrac{dy}{dx} + y\right) + 2y\dfrac{dy}{dx} + 1 + 2\dfrac{dy}{dx} = 0$

$\Rightarrow 4x - 3y + 1 = \dfrac{dy}{dx}(3x - 2y - 2)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{3y - 4x - 1}{2y - 3x + 2}$

115 (c)

We have, $\sin y + e^{-x\cos y} = e$

Differentiating w.r.t. x, we get

$\cos y \dfrac{dy}{dx} - e^{-x\cos y}\left(\cos y - x\sin y\dfrac{dy}{dx}\right) = 0$

Putting $x = 1, y = \pi$, we get

$-\dfrac{dy}{dx} - e(-1) = 0 \Rightarrow \dfrac{dy}{dx} = e$

116 (a)

$$\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right] = \frac{d}{dx}[\tan^{-1}a - \tan^{-1}x]$$

$$= 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$$

117 (b)

$$\because y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$$

$$= \tan^{-1}\left[\tan\left(\tan^{-1}\left(\frac{a}{b}\right) - x\right)\right]$$

$$\Rightarrow y = \tan^{-1}\left(\frac{a}{b}\right) - x$$

$$\therefore \frac{dy}{dx} = 0 - 1 = -1$$

118 (b)

Given, $y = x^y$

$$\Rightarrow \log y = y\log x$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = y\cdot\frac{1}{x} + \frac{dy}{dx}\cdot\log x$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1}{y} - \log x\right] = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 - y\log x)}$$

119 (b)

$$\because x^x y^y z^z = c$$

$$\Rightarrow x\log x + y\log y + z\log z = \log c$$

On differentiating partially w.r.t. x, we get

$$x\cdot\frac{1}{x} + \log x + z\cdot\frac{1}{z}\frac{\partial z}{\partial x} + \log z\frac{\partial z}{\partial x} = 0$$

$$\Rightarrow (1 + \log z)\frac{\partial z}{\partial x} = -(1 + \log x)$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\left(\frac{1 + \log x}{1 + \log z}\right)$$

121 (d)

We have,

$$f(x) = \sqrt{(x-1)^2} = |x-1|$$

$$= \begin{cases} x-1, & \text{if } x \geq 1 \\ -(x-1), & \text{if } x < 1 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } x < 1 \end{cases}$$

122 (a)

$$\because u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$$

$$\therefore \frac{\partial u}{\partial x} = \frac{1}{\sqrt{\left\{1 - \left(\frac{x}{y}\right)^2\right\}}}\frac{1}{y} + \frac{1}{1 + \left(\frac{y}{x}\right)^2}\cdot\left(-\frac{y}{x^2}\right)$$

$$\Rightarrow x\frac{\partial u}{\partial x} = \frac{x}{\sqrt{(y^2 - x^2)}} - \frac{xy}{(x^2 + y^2)} \quad \text{...(i)}$$

and $\dfrac{\partial u}{\partial y} = \dfrac{1}{\sqrt{\left\{1 - \left(\frac{x}{y}\right)^2\right\}}}\left(-\dfrac{x}{y^2}\right) + \dfrac{1}{1 + \left(\frac{y}{x}\right)^2}\cdot\left(\dfrac{1}{x}\right)$

$$\Rightarrow y\frac{\partial u}{\partial y} = -\frac{x}{\sqrt{(y^2 - x^2)}} + \frac{xy}{(x^2 + y^2)} \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

123 (c)

On differentiating the given equation partially w.r.t. x and y respectively

$$u_x = \frac{y}{x} + \log y, \quad u_y = \log x + \frac{x}{y}$$

Now, $u_x u_y - u_x \log x - u_y \log y + \log x \log y$

$$= \left(\frac{y}{x} + \log y\right)\left(\log x + \frac{x}{y}\right) - \left(\frac{y}{x} + \log y \log x\right)$$

$$- \left(\log x + \frac{x}{y}\right)\log y + \log x \log y = 1$$

124 (a)

Here, $x = A\cos 4t + B\sin 4t$

$$\Rightarrow \frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$$

$$\Rightarrow \frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$$

$$\Rightarrow \frac{d^2x}{dt^2} = -16x$$

125 (a)

$$\lim_{x \to a}\frac{xf(a) - af(x)}{x - a}$$

$$= \lim_{x \to a}\frac{xf(a) - af(a) - af(x) + af(a)}{x - a}$$

$$= \lim_{x \to a}\frac{f(a)(x - a) - a[f(x) - f(a)]}{x - a}$$

$$= \lim_{x \to a}\frac{f(a)(x - a)}{x - a} - a\lim_{x \to a}\frac{f(x) - f(a)}{(x - a)}$$

$$= f(a) - af'(a)$$

126 (b)

$$\frac{d}{dx}[x^x + x^a + a^x + a^a]$$

$$= x^x(1 + \log x) + ax^{a-1} + a^x \log a + 0$$

$$= x^x(1 + \log x) + ax^{a-1} + a^x \log a$$

127 (d)

Given, $y = a^x \cdot b^{2x-1}$

$$\Rightarrow \log y = x\log a + (2x - 1)\log b$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \log a + 2\log b$$

$$\Rightarrow \frac{dy}{dx} = y\log ab^2$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx}\log ab^2 = y(\log ab^2)^2$$

129 (d)

Differentiating $ax^2 + 2hxy + by^2 = 1$ w.r.t. x, we get

$$2ax + 2hy + 2hx\frac{dy}{dx} + 2by\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{ax + hy}{hx + by}\right)$$

$$\Rightarrow \frac{d^2y}{dx^2}$$

$$= \left\{\frac{(hx + by)\left(a + h\frac{dy}{dx}\right) - (ax + hy)\left(h + b\frac{dy}{dx}\right)}{(hx + by)^2}\right\}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^3}$$

130 (d)

$[hog](x) = h(x^2) = 2\log_e x$

$\Rightarrow (hogof)(x) = hog(\sin x) = 2\log_e \sin x$

$\Rightarrow F(x) = 2\log_e \sin x$

$\Rightarrow F'(x) = 2\cot x$

$\Rightarrow F''(x) = -2\,\text{cosec}^2\, x$

131 (b)

Since, $\sin^{-1} x = \frac{\pi}{2} - \sin^{-1} y$

$\Rightarrow \sin^{-1} x = \cos^{-1} y$

$\Rightarrow y = \sqrt{1 - x^2}$ $(\because \sin^{-1} x = \cos^{-1}\sqrt{1 - x^2})$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 - x^2}}(-2x) = -\frac{x}{y}$$

132 (d)

$y = \tan^{-1}(\sec x - \tan x)$

$$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{1 - \sin x}{\cos x}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}\right)$$

$$= \frac{d}{dx}\tan^{-1}\left(\frac{1 - \tan x/2}{1 + \tan x/2}\right)$$

$$= \frac{d}{dx}\tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}$$

$$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$$

133 (d)

We have,

$x = e^t \sin t$ and $y = e^t \cos t$

$\Rightarrow \frac{dx}{dt} = e^t(\sin t + \cos t)$ and $\frac{dy}{dt} = e^t(\cos t - \sin t)$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$$

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\cos t - \sin t}{\cos t + \sin t}\right) \times \frac{1}{e^t(\sin t + \cos t)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(\cos t + \sin t)^2 - (\cos t - \sin t)^2}{(\cos t + \sin t)^2}$$

$$\times \frac{1}{e^t(\cos t + \sin t)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2}{(\cos t + \sin t)^3 e^t} \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{t=\pi} = \frac{-2}{-e^\pi}$$

$$= \frac{2}{e^\pi}$$

134 (b)

Given, $f(x) = 3|2 + x|$

$$f(x) = \begin{cases} 3(2 + x), & x \geq -2 \\ -3(2 + x), & x \leq -2 \end{cases}$$

On differentiating w. r. t, x, we get

$$f'(x) = \begin{cases} 3, & x \geq 2 \\ -3, & x \leq -2 \end{cases}$$

at $x = -3, f'(3) = -3$

135 (c)

We know that be Newton's Leibnitz formula

If $I = \int_u^v f(t)dx$,

Then $\frac{dI}{dx} = f(v)\frac{dv}{dx} - f(u)\frac{du}{dx}$

Where u and v are function of x

$$\therefore \frac{dx}{dy} = \frac{1}{\sqrt{1 + 9y^2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{1 + 9y^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{9y}{\sqrt{1 + 9y^2}} \cdot \frac{dy}{dx}$$

$$= \frac{9y}{\sqrt{1 + 9y^2}}\sqrt{1 + 9y^2} = 9y$$

136 (a)

Given, $f(x) = x\tan^{-1} x$

$\therefore f'(x) = \frac{x}{1 + x^2} + \tan^{-1} x$

$\Rightarrow f'(1) = \frac{1}{1 + 1^2} + \tan^{-1} 1 = \frac{1}{2} + \frac{\pi}{4}$

137 (b)

Given, $g(x) = [f(2f(x) + 2)]^2$

$\therefore g'(x) = 2.f(2f(x) + 2).f'(2f(x) + 2).2f'(x)$

$= 4f(2f(x) + 2)f'(2f(x) + 2)f'(x)$

$\therefore g'(0) = 4f(0)f'(0)f'(0) = -4$

138 (b)

Let $f(x) = |x - 1| + |x - 3|$

$$f(x) = \begin{cases} -(x - 1) - (x - 3), & x < 1 \\ (x - 1) - (x - 3), & 1 \leq x < 3 \\ (x - 1) + (x - 3), & x \geq 3 \end{cases}$$

$$f(x) = \begin{cases} 4 - 2x, & x < 1 \\ 2, & 1 \leq x < 3 \\ 2x - 4, & x \geq 3 \end{cases}$$

At $x = 2$,

$f(x) = 2 \Rightarrow f'(x) = 0$

139 (d)

We have, $y = \tan^{-1}(\sec x - \tan x)$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{1 - \sin x}{\cos x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)$$

$$= \frac{d}{dx}\tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$$

140 (a)

Given that, $x = a\cos^4\theta$ and $y = a\sin^4\theta$

On differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = 4a\cos^3\theta(-\sin\theta)$$

and $\frac{dy}{d\theta} = 4a\sin^3\theta\cos\theta$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{4a\sin^3\theta\cos\theta}{4a\cos^3\theta\sin\theta}$$

$$= -\frac{\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$$

Now, $\left(\frac{dy}{dx}\right)_{\theta=\frac{3\pi}{4}} = -\tan^2\left(\frac{3\pi}{4}\right) = -1$

141 (b)

Given, $\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \sec^{-1}e^a$

$$\Rightarrow \frac{\left[\begin{matrix}(x^2+y^2)\left(2x - 2y\frac{dy}{dx}\right)\\ -(x^2-y^2)\left(2x + 2y\frac{dy}{dx}\right)\end{matrix}\right]}{(x^2+y^2)^2} = 0$$

$$\Rightarrow (2x^3 + 2xy^2 - 2x^3 + 2xy^2) - 2y\frac{dy}{dx}$$

$$(x^2 + y^2 + x^2 - y^2) = 0$$

$$\Rightarrow 4xy^2 - 4x^2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

142 (a)

$$\because y = \frac{a + bx^{3/2}}{x^{5/4}}$$

On differentiating w.r.t. x, we get

$$y' = \frac{\frac{3}{2}bx^{7/4} - \frac{5}{4}(a + bx^{3/2})x^{1/4}}{(x^{5/4})^2}$$

$\because y' = 0$ at $x = 5$

$\therefore \frac{3}{2}bx^{7/4} - \frac{5}{4}(a + bx^{3/2})x^{1/4} = 0$, at $x = 5$

$\Rightarrow 6bx^{3/2} - 5(a + bx^{3/2}) = 0$, at $x = 5$

$\Rightarrow bx^{3/2} = 5a$, at $x = 5 \Rightarrow b(5)^{3/2} = 5a$

$$\Rightarrow \frac{a}{b} = \frac{5^{3/2}}{5} \Rightarrow a:b = \sqrt{5}:1$$

143 (c)

Given, $f(x) = be^{ax} + ae^{bx}$

$\Rightarrow f'(x) = abe^{ax} + abe^{bx}$

$\Rightarrow f''(x) = a^2be^{ax} + ab^2e^{bx}$

$\Rightarrow f''(0) = a^2b + ab^2 = ab(a + b)$

144 (b)

Given, $x^m y^n = (x + y)^{m+n}$

$m\log x + n\log y = (m + n)\log(x + y)$

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m + n)}{(x + y)}\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{n}{y} - \frac{(m + n)}{(x + y)}\right] = \frac{m + n}{x + y} - \frac{m}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1, y=2} = 2$$

145 (a)

Since, $f\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$

$$f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$$

$$= \left(x - \frac{1}{x}\right)\left[\left(x - \frac{1}{x}\right)^2 + 3\right]$$

$\Rightarrow f(x) = x(x^2 + 3) = x^3 + 3x$

$\Rightarrow f'(x) = 3x^2 + 3$

146 (b)

We have,

$$x = a\cos t + \frac{b}{2}\cos 2t,\ y = a\sin t + \frac{b}{2}\sin 2t$$

$$\Rightarrow \frac{dy}{dx} = -a\sin t - b\sin 2t,\ \frac{dy}{dt}$$

$$= a\cos t + b\cos 2t$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t + b\cos 2t}{-a\sin t - b\sin 2t}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(a\cos t + b\cos 2t)}{(a\sin t + b\sin 2t)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$$

$$= -\frac{d}{dt}\frac{(a\cos t + b\cos 2t)}{(a\sin t + b\sin 2t)}\cdot\frac{dt}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{a^2 + 2b^2 + 3ab\cos t}{(a\sin t + b\sin 2t)^3}\right]$$

$$\therefore \frac{d^2y}{dx^2} = 0 \Rightarrow a^2 + 2b^2 + 3ab\cos t = 0 \Rightarrow \cos t$$

$$= -\left(\frac{a^2 + 2b^2}{3ab}\right)$$

147 (d)

$h'(x) = [f(x)^2 + g(x)^2]$

$\Rightarrow h''(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$$\left[\begin{matrix}\because g(x) = f'(x)\\ \Rightarrow g'(x) = f''(x)\end{matrix}\right]$$

$\therefore h''(x) = 2f(x)g(x) + 2g(x)(-f(x))$

$[\because f''(x) = -f(x)]$

$\Rightarrow h''(x) = 0$

$\Rightarrow h'(x) = C$, a constant for all $x \in R$

$\Rightarrow h(x) = Cx + C_1$

$\Rightarrow h(0) = C_1$ and $h(1) = C + C_1$

$\Rightarrow 2 = C_1$ and $8 = C + C_1$

$\Rightarrow C_1 = 2$ and $C = 6$

$\therefore h(x) = 6x + 2$

$\Rightarrow h(2) = 6 \times 2 + 2 = 14$

148 (b)

Given, $y = x \log x$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{x}{x} + \log x$

$\Rightarrow \dfrac{dy}{dx} = \log e + \log x$

$\Rightarrow \dfrac{dy}{dx} = \log(ex)$

149 (b)

Given, $f''(x) = -f(x)$

$\Rightarrow g'(x) = -f(x)$ and $f'(x) = g(x)$ ...(i)

Now, $F(x) = \left(f\left(\dfrac{x}{2}\right)\right)^2 + \left(g\left(\dfrac{x}{2}\right)\right)^2$

$\therefore F'(x) = 2\left(f\left(\dfrac{x}{2}\right)\right) \cdot f'\left(\dfrac{x}{2}\right) \cdot \dfrac{1}{2}$

$\quad + 2\left(g\left(\dfrac{x}{2}\right)\right) \cdot g'\left(\dfrac{x}{2}\right) \cdot \dfrac{1}{2} = 0$

[using Eq.(i)]

$\therefore F(x)$ is a constant $\Rightarrow F(10) = F(5) = 5$

150 (a)

$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$

$\Rightarrow y = \sqrt{\sin x + y}$

$\Rightarrow y^2 = \sin x + y$

On differentiating w.r.t. x, we get

$2y \dfrac{dy}{dx} = \cos x + \dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\cos x}{2y - 1}$

151 (a)

Given, $y = x^2 e^{mx} \Rightarrow \dfrac{dy}{dx} = 2x e^{mx} + m x^2 e^{mx}$

$\Rightarrow \dfrac{d^2 y}{dx^2} = 2(e^{mx} + mx e^{mx}) + m(2x e^{mx}$

$\quad\quad - x^2 m e^{mx})$

$\Rightarrow \dfrac{d^2 y}{dx^2} = e^{mx}(m^2 x^2 + 4mx + 2)$

$\Rightarrow \dfrac{d^3 y}{dx^3} = e^{mx}[m^3 x^2 + 4m^2 x + 2m + 2m^2 x + 4m]$

$\quad = e^{mx}[m^3 x^2 + 6m^2 x + 6m]$

152 (d)

Given, $x^2 + y^2 = t - \dfrac{1}{t}$ and $x^4 + y^4 = t^2 + \dfrac{1}{t^2}$

$\Rightarrow x^4 + y^4 + 2x^2 y^2 = t^2 + \dfrac{1}{t^2} - 2$

$\Rightarrow x^4 + y^4 + 2x^2 y^2 = x^4 + y^4 - 2$

$\Rightarrow x^2 y^2 + 1 = 0 \Rightarrow y^2 = \dfrac{-1}{x^2}$

On differentiating w.r.t. x, we get

$2y \dfrac{dy}{dx} = \dfrac{2}{x^3} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x^3 y}$

153 (d)

Given that, $y = \cos^{-1}\sqrt{1 - t^2} = \sin^{-1} t$

and $x = \sin^{-1}(3t - 4t^3) = 3\sin^{-1} t$

On differentiating both w.r.t. t respectively, we get

$\dfrac{dy}{dt} = \dfrac{1}{\sqrt{1 - t^2}}$ and $\dfrac{dx}{dt} = \dfrac{3}{\sqrt{1 - t^2}}$

$\therefore \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\left(\frac{1}{\sqrt{1-t^2}}\right)}{3\left(\frac{1}{\sqrt{1-t^2}}\right)} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3}$

154 (c)

We have,

$f(x) = \sqrt{1 - \sin 2x} = \sqrt{(\cos x - \sin x)^2}$

$\Rightarrow f(x) = |\cos x - \sin x|$

$\Rightarrow f(x) = \begin{cases} \cos x - \sin x, & \text{for } 0 \le x \le \pi/4 \\ -(\cos x - \sin x), & \text{for } \pi/4 < x \le \pi/2 \end{cases}$

$\therefore f'(x) = \begin{cases} -(\cos x - \sin x), & \text{for } 0 < x < \pi/4 \\ (\cos x + \sin x), & \text{for } \pi/4 < x < \pi/2 \end{cases}$

155 (c)

Let $u = \sin x$ and $v = \cos x$

On differentiating w.r.t. x respectively, we get

$\dfrac{du}{dx} = \cos x$ and $\dfrac{dv}{dx} = -\sin x$

$\therefore \dfrac{du}{dv} = \dfrac{du/dx}{dv/dx} = -\cot x$

156 (d)

Let $y = F\{f(\phi(x))\}$

On differentiating w.r.t. x, we get

$y' = F'[f\{\phi(x)\}] \dfrac{d}{dx} f\{\phi(x)\}$

$\quad = F'[f\{\phi(x)\}] f'\{\phi(x)\} \dfrac{d}{dx} \phi(x)$

$\quad = F'[f\{\phi(x)\}] f'\{\phi(x)\} \phi'(x)$

157 (b)

Given, $x\sqrt{1 + y} = -y\sqrt{1 + x}$ ...(i)

On squaring both sides, we get

$x^2(1 + y) = y^2(1 + x)$

$\Rightarrow (x - y)(x + y) + xy(x - y) = 0$

$\Rightarrow (x - y)(x + y + xy) = 0$

$x - y \ne 0$ because it does not satisfy the Eq. (i).

$\therefore x + y + xy = 0 \Rightarrow y = -\dfrac{x}{1 + x}$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{(1 + x)(1) - x(1)}{(1 + x)^2} = -\dfrac{1}{(1 + x)^2}$

158 (b)

$\therefore \dfrac{x^2 - y^2}{x^2 + y^2} = \sec^{-1}(e^a)$

On differentiating w.r.t. x, we get
$$\frac{(x^2 + y^2)\left(2x - 2y\frac{dy}{dx}\right) - (x^2 - y^2)\left(2x + 2y\frac{dy}{dx}\right)}{(x^2 + y^2)^2}$$
$$= 0$$
$$\Rightarrow x(x^2 + y^2) - y(x^2 + y^2)\frac{dy}{dx}$$
$$= (x^2 - y^2)x + y(x^2 - y^2)\frac{dy}{dx}$$
$$\Rightarrow (x^2 y - y^3 + x^2 y + y^3)\frac{dy}{dx}$$
$$= (x^3 + xy^2 - x^3 + xy^2)$$
$$\Rightarrow \frac{dy}{dx} = \frac{2xy^2}{2x^2 y} = \frac{y}{x}$$

159 (a)

We have,
$$f(x) = \log_a(\log_a x)$$
$$\Rightarrow f'(x) = \frac{1}{\log_a x \cdot \log_e a}\frac{d}{dx}(\log_a x)$$
$$\Rightarrow f'(x) = \frac{1}{\log_a x \log_e a} \times \frac{1}{x \log_e a} = \frac{\log_a e}{x \log_e x}$$

160 (b)

$$\because y = \log^n x$$
On differentiating w.r.t. x, we get
$$x \log x \log^2 x \log^3 x \dots \log^{n-1} x \log^n x \frac{dy}{dx}$$
$$= \frac{x \log x \log^2 x \log^3 x \dots \log^{n-1} x \log^n x . 1}{x \log x \log^2 x \log^3 x \dots \log^{n-1} x}$$
$$= \log^n x$$

161 (b)

Let $D = \begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$

$$\Rightarrow D = \begin{vmatrix} \sin px & p \cos px & -p^2 \sin px \\ -p^3 \cos px & p^4 \sin px & p^5 \cos px \\ -p^6 \sin px & -p^7 \cos px & p^8 \sin px \end{vmatrix}$$

Taking p^3 and p^6 common from R_2 and R_3 row
$$= p^9 \begin{vmatrix} \sin px & p \cos px & -p^2 \sin px \\ -\cos px & p \sin px & p^2 \cos px \\ -\sin px & -p \cos px & p^2 \sin px \end{vmatrix}$$

$$= -p^9 \begin{vmatrix} \sin px & p \cos px & -p^2 \sin px \\ -\cos px & p \sin px & p^2 \cos px \\ \sin px & p \cos px & -p^2 \sin px \end{vmatrix}$$

$$= 0 \quad (R_1 \text{ and } R_3 \text{ rows are identical})$$

162 (d)

$$y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$
$$\Rightarrow y = e^{-x}$$
On differentiating w.r.t. x, we get
$$\frac{dy}{dx} = -e^{-x}$$
Again differentiating w.r.t. x, we get

$$\frac{d^2 y}{dx^2} = -e^{-x}(-1) = e^{-x} = y$$

163 (b)

We have, $f(4) = 4$ and $f'(4) = 1$
$$\therefore \lim_{x \to 4} \frac{2 - \sqrt{f(x)}}{2 - \sqrt{x}}$$
$$= \lim_{x \to 4} \frac{\frac{-f'(x)}{2\sqrt{f(x)}}}{-\frac{1}{2\sqrt{x}}} \quad [\text{Using L'Hospital's Rule}]$$
$$\Rightarrow \lim_{x \to 4} \frac{2 - \sqrt{f(x)}}{2 - \sqrt{x}} = \lim_{x \to 4} \frac{\sqrt{x}\, f'(x)}{\sqrt{f(x)}} = \frac{2f'(4)}{\sqrt{f(4)}} = \frac{2}{2}$$
$$= 1$$

164 (c)

$$\because x = \frac{2at}{1 + t^3} \text{ and } y = \frac{2at^2}{(1 + t^3)^2}$$
$$\therefore 2ay = x^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{x}{a}$$

165 (c)

Given, $y = e^{a \sin^{-1} x}$
On differentiating w.r.t. x, we get
$$y_1 = e^{a \sin^{-1} x} a. \frac{1}{\sqrt{1 - x^2}}$$
$$\Rightarrow y_1 \sqrt{1 - x^2} = ay$$
$$\Rightarrow (1 - x^2)y_1^2 = a^2 y^2$$
Again, differentiating w.r.t. x, we get
$$(1 - x^2)2y_1 y_2 - 2xy_1^2 = a^2 2yy_1$$
$$\Rightarrow (1 - x^2)y_2 - xy_1 - a^2 y = 0$$
Using Leibnitz's rule,
$$(1 - x^2)y_{n+2} + {}^nC_1 y_{n+1}(-2x) + {}^nC_2 y_n(-2)$$
$$-xy_{n+1} - {}^nC_1 y_n - a^2 y_n = 0$$
$$\Rightarrow (1 - x^2)y_{n+2} + xy_{n+1}(-2n - 1)$$
$$+ y_n[-n(n - 1) - n - a^2] = 0$$
$$\Rightarrow (1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} = (n^2 + a^2)y_n$$

166 (a)

Since, $\frac{x-y}{x+y} = \sec^{-1} a$
$$\Rightarrow \frac{(x+y)\left(1 - \frac{dy}{dx}\right) - (x-y)\left(1 + \frac{dy}{dx}\right)}{(x+y)^2} = 0$$
$$\Rightarrow x + y - x + y - (x + y + x - y)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}$$
$$= \frac{y}{x}$$

167 (c)

Given, $y = x + x^2 + x^3 + \dots \Rightarrow y = \frac{x}{1-x}$
$$\Rightarrow x = \frac{y}{1 + y} = y - y^2 + y^3 - \dots$$
On differentiating w.r.t. y, we get
$$\frac{dx}{dy} = 1 - 2y + 3y^2 - \dots$$

168 (b)

Let $y = \sqrt{\dfrac{1-\sin 2x}{1+\sin 2x}} = \dfrac{\cos x - \sin x}{\cos x + \sin x}$

$= \dfrac{1 - \tan x}{1 + \tan x} = \tan\left(\dfrac{\pi}{4} - x\right)$

$\Rightarrow \dfrac{dy}{dx} = -\sec^2\left(\dfrac{\pi}{4} - x\right)$

169 (b)

Let $u = \log_{10} x$ and $v = x^2$

$\therefore \dfrac{du}{dx} = \dfrac{\log_{10} e}{x}$ and $\dfrac{dv}{dx} = 2x$

$\therefore \dfrac{du}{dv} = \dfrac{du/dx}{dv/dx} = \dfrac{\log_{10} e}{x}\bigg/ 2x$

$= \dfrac{\log_{10} e}{2x^2}$

170 (d)

$\because y = x \ln\left(\dfrac{x}{a+bx}\right) = x(\ln x - \ln(a+bx))$

or $\left(\dfrac{y}{x}\right) = \ln x - \ln(a+bx)$

On differentiating both sides w.r.t. x, we get

$\left(\dfrac{x\frac{dy}{dx} - y.1}{x^2}\right) = \dfrac{1}{x} - \dfrac{b}{a+bx} = \dfrac{a}{x(a+bx)}$ ... (i)

or $\left(x\dfrac{dy}{dx} - y\right) = \dfrac{ax}{a+bx}$

On taking log on both sides, we get

$\ln\left(x\dfrac{dy}{dx} - y\right) = \ln(ax) - \ln(a+bx)$

On differentiating both sides w.r.t. x, we get

$\dfrac{x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx}}{\left(x\frac{dy}{dx} - y\right)} = \dfrac{1}{x} - \dfrac{b}{a+bx} = \dfrac{a}{x(a+bx)}$

$= \dfrac{\left(x\frac{dy}{dx} - y\right)}{x^2}$ [from Eq. (i)]

or $x^3\dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y\right)^2$

172 (a)

On differentiating given equation w.r.t. x, we get

$4x - 3x\dfrac{dy}{dx} - 3y + 2y\dfrac{dy}{dx} + 1 + 2\dfrac{dy}{dx} - 0 = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{3y - 4x - 1}{2y - 3x + 2}$

173 (c)

Given, $y = \sqrt{\sin x + y}$ $\Rightarrow$ $y^2 = \sin x + y$

$\Rightarrow 2y\dfrac{dy}{dx} = \cos x + \dfrac{dy}{dx}$

$\Rightarrow (2y - 1)\dfrac{dy}{dx} = \cos x$

174 (b)

We have,

$2^x + 2^y = 2^{x+y}$

Differentiating with respect to x, we get

$2^x \log 2 + 2^y \log 2 \dfrac{dy}{dx} = 2^{x+y} \log 2 \left(1 + \dfrac{dy}{dx}\right)$

$\Rightarrow 2^x + 2^y\dfrac{dy}{dx} = 2^{x+y}\left(1 + \dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2^x - 2^{x+y}}{2^{x+y} - 2^y} \Rightarrow \left(\dfrac{dy}{dx}\right)_{(1,0)} = \dfrac{2-4}{4-2} = -1$

175 (a)

Given, $y = \sec(\tan^{-1} x)$

$\Rightarrow y = \sec(\sec^{-1}\sqrt{1+x^2}) = \sqrt{1+x^2}$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1+x^2}}(2x) = \dfrac{x}{\sqrt{1+x^2}}$

176 (b)

Given, $f(x) = (x-7)^2(x-2)^7$

$\Rightarrow f(\theta) = (\theta - 7)^2(\theta - 2)^7$

On differentiating w.r.t. θ, we get

$\Rightarrow f'(\theta) = 2(\theta - 7)(\theta - 2)^7$
$\qquad\qquad + 7(\theta - 2)^6(\theta - 7)^2$

put $f'(\theta) = 0$

$\Rightarrow (\theta - 7)(\theta - 2)^6[2(\theta - 2) + 7(\theta - 7)] = 0$

$\Rightarrow 9\theta = 53 \Rightarrow \theta = \dfrac{53}{9}$

177 (b)

We have,

$x^2 + y^2 = t - \dfrac{1}{t}$ and $x^4 + y^4 = t^2 + \dfrac{1}{t^2}$

$\Rightarrow (x^2 + y^2)^2 = t^2 + \dfrac{1}{t^2} - 2$

$\Rightarrow (x^2 + y^2)^2 = x^4 + y^4 - 2$

$\Rightarrow 2x^2y^2 = -2$

$\Rightarrow x^2y^2 = -1$

$\Rightarrow y^2 = -\dfrac{1}{x^2} \Rightarrow 2y\dfrac{dy}{dx} = \dfrac{2}{x^3} \Rightarrow x^3y\dfrac{dy}{dx} = 1$

178 (b)

Let $f(x) = |x-1| + |x-5|$

$\Rightarrow f(x) = \begin{cases} -2x+6, & x < 1 \\ 4 & 1 \le x < 5 \\ 2x-6 & x \ge 5 \end{cases}$

$\therefore \dfrac{d}{dx}(f(x)) = \begin{cases} -2, & x < 1 \\ 0 & 1 < x < 5 \\ 2 & x > 5 \end{cases}$

Hence, $\left(\dfrac{d}{dx}(f(x))\right)_{x=3} = 0$

179 (b)

$\sin^{-1} x + \sin^{-1} y = \dfrac{\pi}{2}$

$\Rightarrow \sin^{-1} x = \dfrac{\pi}{2} - \sin^{-1} y$

$\Rightarrow \sin^{-1} x = \cos^{-1} y$

$\Rightarrow y = \sqrt{1 - x^2}$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{1-x^2}}(-2x) = -\dfrac{x}{y}$

180 (c)

$$f[f(f(x))] = f\left[f\left(\frac{1}{1-x}\right)\right]$$

$$= f\left(\frac{1}{1-\frac{1}{1-x}}\right)$$

$$\left[\because f(x) = \frac{1}{1-x}\right]$$

$$= f\left(\frac{1-x}{-x}\right) = \frac{1}{1+\left(\frac{1-x}{x}\right)}$$

$$\Rightarrow f[f(f(x))] = x$$

∴ The derivative of composite function is equal to 1.

181 (a)

Given that, $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$

Taking log on both sides, we get

$$\log x = \tan^{-1}\left(\frac{y-x^2}{x^2}\right)$$

$$\Rightarrow \frac{y-x^2}{x^2} = \tan(\log x)$$

$$\Rightarrow y = x^2 \tan(\log x) + x^2$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 2x\tan(\log x) + x^2 \frac{\sec^2(\log x)}{x} + 2x$$

$$\Rightarrow \frac{dy}{dx} = 2x\tan(\log x) + x\sec^2(\log x) + 2x$$

$$\Rightarrow \frac{dy}{dx} = 2x[1 + \tan(\log x)] + x\sec^2(\log x)$$

182 (b)

Given, $x = \frac{\sin y}{\sin(a+y)}$

$$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin\cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

183 (b)

Given, $f(x) = e^x \sin x$

$$\Rightarrow f'(x) = e^x \cos x + \sin x e^x$$

$$\Rightarrow f''(x) = e^x \cos x - e^x \sin x$$
$$+ e^x \sin x + e^x \cos x$$

$$= 2e^x \cos x$$

184 (b)

We know that,

$$\cos A \cos 2A \cos 2^2 A \dots \cos 2^{n-1} A = \frac{\sin(2^n A)}{2^n \sin A}$$

$$\therefore \cos x \cos 2x \cos 4x \cos 8x \cos 16x = \frac{\sin 32x}{32 \sin x}$$

$$\Rightarrow f(x) = \frac{1}{32} \cdot \frac{\sin 32x}{\sin x}$$

$$\therefore f'(x) = \frac{1}{32} \times \frac{\sin x(32\cos 32x) - \sin 32x \cos x}{\sin^2 x}$$

$$\Rightarrow f'\left(\frac{\pi}{4}\right) = \frac{1}{\sin\frac{\pi}{4}} = \sqrt{2}$$

185 (a)

Given, $\frac{1+x}{1-y} = \sec a$

$$\Rightarrow y\sec a = \sec a - 1 - x$$

$$\Rightarrow \frac{dy}{dx}\sec a = -1$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sec a} = \frac{-1}{\left(\frac{1+x}{1-y}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-1}{x+1}$$

186 (c)

Let $u = a^{\sec x}$ and $v = a^{\tan x}$

$$\Rightarrow \frac{du}{dx} = a^{\sec x} \log_e a. \sec x \tan x$$

and $\frac{dv}{dx} = a^{\tan x} \log_e a. \sec^2 x$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{a^{\sec x} \log_e a \sec x \tan x}{a^{\tan x} \log_e a \sec^2 x}$$

$$= a^{\sec x - \tan x} \sin x$$

187 (b)

On differentiating given curves w.r.t. θ respectively, we get

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\left(\frac{\theta}{2}\right)} \cdot \sec^2\frac{\theta}{2} \cdot \frac{1}{2}\right)$$

and $\frac{dy}{d\theta} = a\cos\theta$

$$\Rightarrow \frac{dx}{d\theta} = \frac{a\cos^2\theta}{\sin\theta} \text{ and } \frac{dy}{d\theta} = a\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{a\cos^2\theta/\sin\theta} = \tan\theta$$

188 (c)

Since, $\phi(x) = f^{-1}(x) \Rightarrow x = f\{\phi(x)\}$

On differentiating w.r.t. x, we get

$$1 = f'\{\phi(x)\}. \phi'(x)$$

$$\Rightarrow \phi'(x) =$$

$$\frac{1}{f'\{\phi(x)\}} \qquad \dots(i)$$

But $f'\{\phi(x)\} = \frac{1}{1+\{\phi(x)\}^5}$ $\left(\because f'(x) = \frac{1}{1+x^5}\right)$

∴ From Eq. (i),

$$\phi'(x) = \frac{1}{f'\{\phi(x)\}} = 1 + \{\phi(x)\}^5$$

189 (b)

We have,

$$f(x) = 3 e^{x^2} \Rightarrow f'(x) = 6x e^{x^2}$$

$$\therefore f(0) = 3 \text{ and } f'(0) = 0$$

Now,

$$f'(x) - 2x\,f(x) + \frac{1}{3}f(0) - f'(0)$$
$$= 6x\,e^{x^2} - 6x\,e^{x^2} + \frac{1}{3}(3) - 0$$
$$= 1$$

190 (c)

On differentiating partially the given equation w.r.t. x and y

$$\frac{\partial u}{\partial x} = \frac{(x+y)}{(x^2+y^2)} \times \frac{x^2-y^2+2xy}{(x+y)^2}$$

$$\Rightarrow x\frac{\partial u}{\partial x} = \frac{x}{(x^2+y^2)} \times \frac{x^2-y^2+2xy}{(x+y)} \qquad \text{...(i)}$$

and $\frac{\partial u}{\partial y} = \frac{(x+y)}{(x^2+y^2)} \times \frac{y^2-x^2+2xy}{(x+y)^2}$

$$\Rightarrow y\frac{\partial u}{\partial y} = \frac{y}{(x^2+y^2)} \times \frac{(y^2-x^2+2xy)}{(x+y)} \qquad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{(x^2+y^2)(x+y)}$$
$$[x(x^2-y^2+2xy) + y(y^2-x^2+2xy)]$$
$$= \frac{1}{(x^2+y^2)(x+y)} \times (x^2+y^2)(x+y)$$
$$= 1$$

191 (a)

We have,

$$F(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt$$

$$\Rightarrow x^2 F(x) = \int_4^x (4t^2 - 2F'(t))\,dt$$

Differentiating both sides with respect to x, we get

$$2x\,F(x) + x^2 F'(x) = 4x^2 - 2F'(x)$$

Putting $x = 4$, we get

$$8F(4) + 16F'(4) = 64 - 2F'(4)$$
$$\Rightarrow 18F'(4) = 64 \quad [\because F(4) = 0]$$
$$\Rightarrow F'(4) = \frac{32}{9}$$

192 (d)

Put $x^2 = \cos 2\theta$ in the given equation, we get

$$y = \tan^{-1}\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}$$

$$= \tan^{-1}\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right)$$

$$\Rightarrow y = \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2$$

$$\Rightarrow \frac{dy}{dx} = 0 - \frac{1}{2}\left(-\frac{(2x)}{\sqrt{1-x^4}}\right) = \frac{x}{\sqrt{1-x^4}}$$

193 (b)

We have,

$$f(x) = |x^2 - 5x + 6|$$

$$\Rightarrow f(x) = \begin{cases} x^2 - 5x + 6, & \text{if } x \geq 3 \text{ or } x \leq 2 \\ -(x^2 - 5x + 6), & \text{if } 2x < x < 3 \end{cases}$$

$$\therefore f'(x) = \begin{cases} (2x - 5), & \text{if } x > 3 \text{ or } x < 2 \\ -(2x - 5), & \text{if } 2 < x < 3 \end{cases}$$

195 (c)

Let $y = x^6 + 6^x$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 6x^5 + 6^x \log 6$$

196 (b)

We have,

$$f(x) = \cos^2 x + \cos^2(x + \pi/3) + \sin x \sin(x + \pi/3)$$

$$\Rightarrow f(x) = \frac{1}{2}\left[1 + \cos 2x + 1 + \cos\left(2x + \frac{2\pi}{3}\right) + \cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right]$$

$$\Rightarrow f(x) = \frac{1}{2}\left[\frac{5}{2} + \cos 2x + \cos\left(2x + \frac{2\pi}{3}\right) - \cos\left(2x + \frac{\pi}{3}\right)\right]$$

$$\Rightarrow f(x) = \frac{1}{2}\left[\frac{5}{2} + 2\cos\left(2x + \frac{\pi}{3}\right)\cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right]$$

$$\Rightarrow f(x) = \frac{5}{4}$$

$$\therefore gof(x) = g(5/4) = 3 \text{ for all } x$$

$$\Rightarrow \frac{d}{dx}(gof(x)) = 0 \text{ for all } x$$

197 (c)

We have,

$$y = \sin^{-1}x + \cos^{-1}y \Rightarrow y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$$

198 (b)

Given, $z = y + f(v)$ \quad ...(i)

Where $v = \left(\frac{x}{y}\right)$

On differentiating partially Eq. (i) w.r.t. x, we get

$$\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right) \cdot \left(\frac{1}{y}\right)$$

$$\Rightarrow v\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right) \cdot \left(\frac{1}{y}\right)\left(\frac{x}{y}\right) = f'\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right) \quad \text{...(ii)}$$

Now, differentiating partially Eq. (i) w.r.t. y, we get

$$\frac{\partial z}{\partial y} = 1 + f'\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right) \qquad \text{...(iii)}$$

On adding Eqs. (ii) and (iii), we get

$$v\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = f'\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right) + 1 + f'\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right) = 1$$

199 (d)

$$y = \tan^{-1}\left(\frac{\sqrt{x} - x}{1 + x^{3/2}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{x} - x}{1 + \sqrt{x}.x}\right)$$

$$= \tan^{-1}(\sqrt{x}) - \tan^{-1}(x)$$

On differentiating w.r.t. x, we get

$$y' = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$$

$$\Rightarrow y'(1) = \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} = -\frac{1}{4}$$

200 (c)

We have,

$$f(1) = 1 \text{ and } f'(1) = 2$$

$$\therefore \lim_{x \to 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$$

$$= \lim_{x \to 1} \frac{\frac{f'(x)}{2\sqrt{f(x)}}}{\frac{1}{2\sqrt{x}}} = \lim_{x \to 1} \frac{\sqrt{x}\, f'(x)}{\sqrt{f(x)}} = \frac{f'(1)}{\sqrt{f(1)}} = 2$$

201 (a)

Let

$$u = \cos^3 x, v = \sin^3 x$$

$$\frac{du}{dx} = -3\cos^2 x \sin x, \frac{dv}{dx} =$$

$$3 \sin^2 x \cos x$$

Now,

$$\frac{du}{dv} = \frac{-3\cos^2 x \sin x}{3\sin^2 x \cos x} =$$

$$-\cot x$$

202 (b)

We have,

$$y = \log\left(\frac{1+x}{1-x}\right)^{1/4} - \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow y = \frac{1}{4}\log(1+x) - \frac{1}{4}\log(1-x) - \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{4(1+x)} + \frac{1}{4(1-x)} - \frac{1}{2(1+x^2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1-x^2)} - \frac{1}{2(1+x^2)} = \frac{x^2}{1-x^4}$$

203 (a)

Given, $y = \left(x + \sqrt{1+x^2}\right)^n$

$$\frac{dy}{dx} = n\left[x + \sqrt{1+x^2}\right]^{n-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{n[x + \sqrt{1+x^2}]^n}{\sqrt{1+x^2}}$$

$$\Rightarrow (1+x^2)\left(\frac{dy}{dx}\right)^2 = n^2 y^2$$

$$\Rightarrow 2\frac{dy}{dx}\frac{d^2y}{dx^2}(1+x^2) + 2x\left(\frac{dy}{dx}\right)^2 = 2n^2 y \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2}(1+x^2) + x\frac{dy}{dx} = n^2 y$$

204 (d)

Let $y = \tan^{-1}\left\{\frac{3\sqrt{x} - x^{3/2}}{1 - 3x}\right\}$

Again let $\sqrt{x} = \tan t$

$$\therefore y = \tan^{-1}\left\{\frac{3\tan t - \tan^3 t}{1 - 3\tan^2 t}\right\} = \tan^{-1}(\tan 3t)$$

$$\Rightarrow y = 3\tan^{-1}\sqrt{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+x} \cdot \frac{1}{2\sqrt{x}} = \frac{3}{2(1+x)\sqrt{x}}$$

205 (d)

We have, $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$

$$\Rightarrow y = \tan^{-1}\left[\frac{1 + \tan x}{1 - \tan x}\right]$$

$$\Rightarrow y = \tan^{-1}\left[\frac{\tan\left(\frac{\pi}{4}\right) + \tan x}{1 - \tan\left(\frac{\pi}{4}\right)\tan x}\right]$$

$$\Rightarrow y = \tan^{-1}\tan\left(\frac{\pi}{4} + x\right)$$

$$\Rightarrow y = \left(\frac{\pi}{4}\right) + x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 1$$

206 (d)

Given, $x = \cos\theta, \quad y = \sin 5\theta$

$$\Rightarrow \frac{dx}{d\theta} = -\sin\theta, \quad \frac{dy}{d\theta} = 5\cos 5\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{5\cos 5\theta}{\sin\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right) \cdot \frac{d\theta}{dx}$$

$$= \frac{d}{d\theta}\left(\frac{-5\cos 5\theta}{\sin\theta}\right)\frac{1}{-\sin\theta}$$

$$= \left(\frac{\sin\theta\sin 5\theta.25 + 5\cos 5\theta\cos\theta}{\sin^2\theta}\right) \cdot \frac{1}{-\sin\theta}$$

$$= -\frac{25\sin 5\theta}{\sin^2\theta} - \frac{5\cos 5\theta\cos\theta}{\sin^3\theta}$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$$

$$= (1 - \cos^2\theta)\left(\frac{-25\sin 5\theta}{\sin^2\theta} - \frac{5\cos 5\theta\cos\theta}{\sin^3\theta}\right)$$

$$- \cos\theta\left(\frac{-5\cos 5\theta}{\sin\theta}\right)$$

$$= \sin^2\theta\left(\frac{-25\sin 5\theta}{\sin^2\theta} - \frac{5\cos\theta\cos 5\theta}{\sin^3\theta}\right)$$

$$+ \frac{5\cos\theta\cos 5\theta}{\sin\theta}$$

$$= -25\sin 5\theta - \frac{5\cos\theta\cos 5\theta}{\sin\theta} + \frac{5\cos\theta\cos 5\theta}{\sin\theta}$$

$$= -25y$$

207 (d)

$$\because f(x) = a + bx$$

$$f\{f(x)\} = a + b(a + bx)$$

$$= ab + a + b^2 x = a(1+b) + b^2 x$$

$$f[f\{f(x)\}] = f\{a(1+b) + b^2 x\}$$

$= a + b\{a(1 + b) + b^2 x\}$

$= a(1 + b + b^2) + b^3 x$

$\therefore f'(x) = a(1 + b + b^2 + \ldots + b^{r-1}) + b^r x$

$= a\left(\dfrac{b^r - 1}{b - 1}\right) + b^r x$

208 (c)

We have,

$x^y = e^{x-y} \Rightarrow y \log x = (x - y) \Rightarrow y = \dfrac{x}{1 + \log x}$

Differentiating w.r.t. x, we get $\dfrac{dy}{dx} = \dfrac{\log x}{(1 + \log x)^2}$

209 (d)

Let $u = \sin^2 x$ and $v = \cos^2 x$

On differentiating w.r.t. x, we get

$\dfrac{du}{dx} = 2 \sin x \cos x = \sin 2x$

and $\dfrac{dv}{dx} = -2 \cos x \sin x = -\sin 2x$

$\therefore \dfrac{du}{dv} = \dfrac{du/dx}{dv/dx} = \dfrac{\sin 2x}{-\sin 2x} = -1$

210 (a)

We have,

$x^p y^q = (x + y)^{p+q}$

$\Rightarrow p \log x + q \log y = (p + q) \log(x + y)$

Diff w.r.t. x, we get

$\dfrac{p}{x} + \dfrac{q}{y} \dfrac{dy}{dx} = \dfrac{p+q}{x+y}\left(1 + \dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dy}{dx}\left(\dfrac{q}{y} - \dfrac{p+q}{x+y}\right) \Rightarrow \dfrac{p+q}{x+y} - \dfrac{p}{x} \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$

212 (a)

Let $y = \sin^{-1}\left(\dfrac{\sqrt{1+x} + \sqrt{1-x}}{2}\right)$

Putting $x = \cos\theta$, we get

$y = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\cos\dfrac{\theta}{2} + \dfrac{1}{\sqrt{2}}\sin\dfrac{\theta}{2}\right)$

$\qquad = \sin^{-1}\left\{\sin\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)\right\}$

$\Rightarrow y = \dfrac{\pi}{4} + \dfrac{1}{2}\theta = \dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}x$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{2\sqrt{1-x^2}}$

213 (c)

Let $y = \tan^{-1}\left(\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

Put $x = \cos 2\theta$

$\therefore \quad y = \tan^{-1}\left(\dfrac{\sqrt{1 + \cos 2\theta} - \sqrt{1 - \cos 2\theta}}{\sqrt{1 + \cos 2\theta} + \sqrt{1 - \cos 2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}\right)$

$= \tan^{-1}\left(\dfrac{1 - \tan\theta}{1 + \tan\theta}\right) = \tan^{-1}\left\{\tan\left(\dfrac{\pi}{4} - \theta\right)\right\}$

$= \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = 0 + \dfrac{1}{2}\cdot\dfrac{1}{\sqrt{1-x}} = \dfrac{1}{2\sqrt{1-x^2}}$

214 (c)

$y = \tan^{-1}x + \cot^{-1}x + \sec^{-1}x + \mathrm{cosec}^{-1}x$

$= \dfrac{\pi}{2} + \dfrac{\pi}{2} = \pi$

$\dfrac{dy}{dx} = 0$

215 (c)

$\because y = \left(\dfrac{ax + b}{cx + d}\right)$

or $cxy + dy = ax + b$

On differentiating both sides w.r.t. x, we get

$c\left\{x\dfrac{dy}{dx} + y.1\right\} + d\dfrac{dy}{dx} = a$

or $x\dfrac{dy}{dx} + y + \left(\dfrac{d}{c}\right)\dfrac{dy}{dx} = \left(\dfrac{a}{c}\right)$

Again differentiating both sides w.r.t. x, we get

$x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + \dfrac{dy}{dx} + \left(\dfrac{d}{c}\right)\dfrac{d^2y}{dx^2} = 0$

or $x + \dfrac{2\frac{dy}{dx}}{\left(\frac{d^2y}{dx^2}\right)} + \dfrac{d}{c} = 0$

Again, on differentiating both sides w.r.t. x, we get

$1 + \dfrac{\left(\frac{d^2y}{dx^2}.2\frac{d^2y}{dx^2} - 2\frac{dy}{dx}.\frac{d^3y}{dx^3}\right)}{\left(\frac{d^2y}{dx^2}\right)^2} + 0 = 0$

$\Rightarrow 2\dfrac{dy}{dx}.\dfrac{d^3y}{dx^3} = 3\left(\dfrac{d^2y}{dx^2}\right)^2$

216 (d)

Given, $y = (\log_{\cos x}\sin x)(\log_{\sin x}\cos x) +$

$\sin^{-1}\dfrac{2x}{1+x^2}$

At $x = \dfrac{\pi}{2}, \log_{\sin x}\cos x$ is not defined.

Hence, we cannot determined the derivative at

$x = \dfrac{\pi}{2}$.

217 (b)

$y = \tan^{-1}\left(\dfrac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

$= \tan^{-1}\left(\dfrac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$

$\qquad\qquad = \tan^{-1}\left[\tan\left\{\tan^{-1}\left(\dfrac{a}{b}\right) - x\right\}\right]$

$\Rightarrow y = \tan^{-1}\left(\dfrac{a}{b}\right) - x$

$\therefore \dfrac{dy}{dx} = 0 - 1 = -1$

218 (d)

We have, $x\cos\theta$, $y = \sin 5\theta$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{5\cos 5\theta}{\sin\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -5\frac{d}{d\theta}\left(\frac{\cos 5\theta}{\sin\theta}\right)\cdot\frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-25\sin\theta\sin 5\theta - 5\cos\theta\cos 5\theta}{\sin^3\theta}$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -25\sin 5\theta = -25y$$

219 (c)

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

$$\therefore y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)$$

$$\Rightarrow y = \frac{\pi}{4} - \theta$$

$$\Rightarrow y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{1}{2\sqrt{1-x^2}}$$

220 (b)

$\because f(x) = (x-2)(x-4)(x-6)\ldots(x-2n)$

Taking log on both sides in the given equation, we get

$\log f(x) = \log(x-2) + \log(x-4) + \ldots + \log(x-2n)$

on differentiating w.r.t. x, we get

$$\frac{1}{f(x)}f'(x) = \frac{1}{(x-2)} + \frac{1}{(x-4)} + \ldots + \frac{1}{(x-2n)}$$

$$\Rightarrow f'(x) = (x-4)(x-6)\ldots(x-2n)$$
$$+(x-2)(x-6)\ldots(x-2n)$$
$$+\cdots+(x-2)(x-6)\ldots(x-2(n-1))$$

$$\therefore f'(2) = (-2)(-4)\ldots(2-2n)$$
$$= (-2)^{n-1}(1.2\ldots(n-1)) = (-2)^{n-1}(n-1)!$$

221 (a)

Given, $x = 2\cos\theta - \cos 2\theta$

and $y = 2\sin\theta - \sin 2\theta$

$$\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

and $\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$$\therefore \frac{dy}{dx} = \frac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta}$$

$$= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$

$$= \frac{2\sin\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)}{2\cos\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)} = \tan\frac{3\theta}{2}$$

222 (c)

Since, $f'(x) > \phi'(x)$

$\Rightarrow 2^{2x-1}2\log 2 > -2^x\log 2 + 2\log 2$

$\Rightarrow 2^{2x} > -2^x + 2$

$\Rightarrow 2^{2x} + 2^x - 2 > 0$

$\Rightarrow (2^x - 1)(2^x + 2) > 0$

$\Rightarrow 2^x - 1 > 0 \qquad [\because 2^x + 2 > 0 \text{ for all } x]$

$\Rightarrow 2^x > 1$

$\therefore x > 0$

223 (b)

We have,

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 = -x^2y + xy^2$

$\Rightarrow (x-y)(x+y) = -xy(x-y)$

$\Rightarrow x + y = -xy$

$$\Rightarrow y = -\frac{x}{1+x} \Rightarrow \frac{dy}{dx} = -\left\{\frac{(1+x)-x}{(1+x)^2}\right\}$$

$$= -\frac{1}{(1+x)^2}$$

224 (c)

$\because y = e^{(1/2)\log(1+\tan^2 x)}$

$\Rightarrow y = (\sec^2 x)^{1/2} = \sec x$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \sec x\tan x$$

225 (a)

Given,

$$f(x) = \frac{1}{4}\left[\frac{x-1}{1} + \frac{(x-1)^3}{3} + \frac{(x-1)^5}{5} + \frac{(x-1)^7}{7} + \cdots\right]$$

$$\Rightarrow f(x) = \frac{1}{4}\left[\frac{1}{2}\log\left(\frac{1+(x-1)}{1-(x-1)}\right)\right] = \frac{1}{8}\log\left(\frac{x}{2-x}\right)$$

$$\Rightarrow f'(x) = \frac{1}{8}\times\frac{1}{\left(\frac{x}{2-x}\right)}\left[\frac{(2-x)1 - x(-1)}{(2-x)^2}\right]$$

$$= \frac{1}{4x(2-x)}$$

226 (d)

$$f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$\Rightarrow f(x) = x^3(-6p^3 - 4p^2) - x^2(p^3 - 4p)$
$\qquad\qquad + 3x^2(p^2 + 6p)$
$\Rightarrow f(x) = -6p^3x^3 - 4p^2x^3 - x^2p^3 + 4px^2$
$\qquad\qquad + 3p^2x^2 + 18px^2$

On differentiating w.r.t. x, we get
$\dfrac{d}{dx}f(x) = -18p^3x^2 - 12p^2x^2 - 2xp^3 + 8px$
$\qquad\qquad + 6p^2x + 36px$

Again differentiating w.r.t. x, we get
$\dfrac{d^2}{dx^2}f(x) = -36p^3x - 24p^2x - 2p^3 + 8p + 6p^2$
$\qquad\qquad + 36p$

Again differentiating w.r.t. x, we get
$\dfrac{d^3}{dx^3}f(x) = -36p^3 - 24p^2 =$ constant

227 (d)

We have,

$f(x) = \arctan\left(\dfrac{x^x - x^{-x}}{2}\right)$

$\Rightarrow f(x) = \tan^{-1}\left\{\dfrac{x^{2x} - 1}{2x^x}\right\}$

$\Rightarrow f(x) = -\tan^{-1}\left\{\dfrac{1 - x^{2x}}{2x^x}\right\}$

$\Rightarrow f(x) = -\cot^{-1}\left\{\dfrac{2x^x}{1 - x^{2x}}\right\}$

$\Rightarrow f(x) = \dfrac{-\pi}{2} + \tan^{-1}\left\{\dfrac{2x^x}{1 - x^{2x}}\right\}$

$\Rightarrow f(x) = \begin{cases} \dfrac{-\pi}{2} + 2\tan^{-1}(x^x), & \text{if } 0 < x < 1 \\ \dfrac{-\pi}{2} - \pi + 2\tan^{-1}(x^x), & \text{if } x > 1 \end{cases}$

$\Rightarrow f(x) = \begin{cases} \dfrac{-\pi}{2} + 2\tan^{-1}(x^x), & \text{if } 0 < x < 1 \\ \dfrac{-3\pi}{2} + 2\tan^{-1}(x^x), & \text{if } x > 1 \end{cases}$

$\Rightarrow f'(x) = \dfrac{2}{1 + x^{2x}} \times x^x(1 + \log_e x)$ for all $x >$
$0, x \neq 1$

Clearly, $f'(1)$ does not exist

228 (a)

We have,
$f(x) + f(y) + f(x)f(y) = 1$ for all $x, y \in R$ ...(i)

Putting $x = y = 0$, we get
$2f(0) + \{f(0)\}^2 = 1$

$\Rightarrow \{f(0)\}^2 + 2f(0) - 1 = 0$

$\Rightarrow f(0) = \dfrac{-2 \pm \sqrt{4 + 4}}{2}$

$\Rightarrow f(0) = -1 \pm \sqrt{2}$

$\Rightarrow f(0) = \sqrt{2} - 1 \quad [\because f(x) > 0 \text{ for all } x]$

Putting $y = x$ in (i), we get
$\{f(x)\}^2 + 2f(x) - 1 = 0$ for all x

$\Rightarrow 2f(x)f'(x) + 2f'(x) = 0$ for all x

$\Rightarrow 2\{f(x) + 1\}f'(x) = 0$ for all x

$\Rightarrow f'(x) = 0$ for all $x[\because f(x) > 0$ for all $x]$

229 (b)

Given, $f(x) = e^x$, $g(x) = \sin^{-1} x$

Since, $h(x) = f[g(x)] = e^{\sin^{-1} x}$

Now, $h'(x) = e^{\sin x} \cdot \dfrac{1}{\sqrt{1 - x^2}}$

$\Rightarrow h'(x) = h(x) \cdot \dfrac{1}{\sqrt{1 - x^2}}$

$\Rightarrow \dfrac{h'(x)}{h(x)} = \dfrac{1}{\sqrt{1 - x^2}}$

230 (b)

$\because f(x, y) = \dfrac{\cos(x - 4y)}{\cos(x + 4y)}$

$\therefore f\left(x, \dfrac{\pi}{2}\right) = \dfrac{\cos(x - 2\pi)}{\cos(x + 2\pi)} = \dfrac{\cos x}{\cos x} = 1$

$\therefore \dfrac{\partial f}{\partial x} = 0$

231 (d)

Given, $y = \sin^n x \cos nx$

$\dfrac{dy}{dx} = n\sin^{n-1} x \cos x \cos nx - n\sin^n x \sin nx$

$= n\sin^{n-1} x[\cos x \cos nx - \sin x \sin nx]$

$= n\sin^{n-1} x \cos(n + 1)x$

232 (a)

Let $f(x) = 3e^{2x}$

Now, $f'(x) = 6e^{2x} = 2f(x)$

Therefore, our assumption is true.

$\therefore (2) = 3e^{2 \times 2} = 3e^4$

233 (c)

$y^2 = P(x) \Rightarrow 2yy' = P'(x)$ ...(i)

$\Rightarrow (2y)y'' + y'(2y') = P''(x)$

$\Rightarrow 2yy'' = P''(x) - 2(y')^2$

$\Rightarrow 2y^3y'' = y^2P''(x) - 2(yy')^2$

$= y^2P''(x) - 2\dfrac{\{P'(x)\}^2}{4}$

[from Eq.(i)]

$\Rightarrow 2y^3 \cdot y'' = P(x)P''(x) - \dfrac{1}{2}\{P'(x)\}^2$

$\therefore \dfrac{d}{dx}(2y^3 \cdot y'')$

$= P(x)P'''(x) + P''(x)P'(x) - P'(x)P''(x)]$

$= P(x) \cdot P'''(x)$

$\Rightarrow 2\dfrac{d}{dx}\left(y^3 \dfrac{d^2y}{dx^2}\right) = P(x)P'''(x)$

234 (c)

Given, $x = e^{y+x}$

$\Rightarrow \log x = (y + x) \Rightarrow \dfrac{1}{x} = \dfrac{dy}{dx} + 1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1 - x}{x}$

235 (a)

Let $y = a\sin^3 t$ and $x = a\cos^3 t$, then
On differentiating w.r.t. t, we get
$$\frac{dy}{dt} = 3a\sin^2 t\cos t$$
and $\frac{dx}{dt} = 3a\cos^2 t(-\sin t)$
$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3a\sin^2 t\cos t}{3a\cos^2 t(-\sin t)} = -\tan t$$
Again differentiating w.r.t. x, we get
$$\frac{d^2 y}{dx^2} = -\sec^2 t\,\frac{dt}{dx} = \frac{-\sec^2 t}{3a\cos^2 t(-\sin t)}$$
$$= \frac{1}{3a}\left(\frac{\sec^4 t}{\sin t}\right)$$
$$\therefore \left(\frac{d^2 y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3a}\cdot\frac{4}{\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{3a}$$

236 (c)
Let $u = \sin x^3$ and $v = \cos x^3$.
On differentiating w.r.t. x, we get
$$\frac{du}{dx} = \cos x^3.3x^2 \text{ and } \frac{dv}{dx} = -\sin x^3.3x^2$$
$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{3x^2\cos x^3}{-3x^2\sin x^3} = -\cot x^3$$

237 (d)
$$\frac{\cos x}{1+\sin x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}$$
$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} = \frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}$$
$$= \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$
$$\therefore = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) = \frac{\pi}{4} - \frac{x}{2}$$
$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$$

238 (a)
Given, $= \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$
$$= \tan^{-1}\sqrt{\frac{1 - \cos\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)}}$$
$$= \tan^{-1}\left|\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right|$$
$$= \frac{\pi}{4} - \frac{x}{2} \qquad \left[\because x = \frac{\pi}{6}\right]$$
$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$$

239 (a)
Given, $y = \frac{\log\sin x}{\log\cos x}$
$$\Rightarrow \frac{dy}{dx} = \frac{\cot x\log\cos x + \tan x\log\sin x}{(\log\cos x)^2}$$

240 (d)
Given, $y = 2^x.3^{2x-1}$
$$\Rightarrow \frac{dy}{dx} = 2^x.3^{2x-1}2\log 3 + 3^{2x-1}.2^x\log 2$$
$$= 2^x 3^{2x-1}\log 18 = y\log 18$$
$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{dy}{dx}\log 18$$
$$= y(\log 18)^2$$

241 (b)
$$\because \quad y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$$
$$= \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$$
$$\Rightarrow y = \frac{\pi}{2} \quad \Rightarrow \frac{dy}{dx} = 0$$

242 (a)
Since, $f(x) = e^{g(x)}$
$$\Rightarrow e^{g(x+1)} = f(x+1) = xf(x) = xe^{g(x)}$$
and $\quad g(x+1) = \log x + g(x)$
$$\Rightarrow g(x+1) - g(x) = \log x \quad ...(i)$$
Replacing x by $x - \frac{1}{2}$, we get
$$g\left(x + \frac{1}{2}\right) - g\left(x - \frac{1}{2}\right) = \log\left(x - \frac{1}{2}\right)$$
$$= \log(2x - 1) - \log 2$$
$$\therefore g''\left(x + \frac{1}{2}\right) - g''\left(x - \frac{1}{2}\right) = -\frac{4}{(2x-1)^2} \quad ...(ii)$$
On substituting, $x = 1,2,3,\ldots N$ in Eq. (ii) and adding, we get
$$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right)$$
$$= -4\Big\{1 + \frac{1}{9}$$
$$+ \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\Big\}$$

243 (b)
We have,
$$y = \log_{x^2+4}(7x^2 - 5x + 1) = \frac{\log_e(7x^2 - 5x + 1)}{\log_e(x^2 + 4)}$$
$$= \frac{\log_e f(x)}{\log_e g(x)}$$
$$\therefore \frac{dy}{dx} = \frac{\log_e\left(g(x)\cdot\frac{f'(x)}{f(x)}\right) - \log_e f(x)\cdot\frac{g'(x)}{g(x)}}{(\log_e g(x))^2}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log_e g(x)}\left\{\frac{f'(x)}{f(x)} - y\frac{g'(x)}{g(x)}\right\}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log_e(x^2+4)}\left\{\frac{14x-5}{7x^2-5x+1} - \frac{2xy}{x^2+4}\right\}$$

245 (a)
We have,
$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right), \text{ where } x = \tan\theta$$

$$\Rightarrow y = \tan^{-1}\left(\frac{1 - \cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\theta$$

$$= \frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1 + x^2)} \Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{2}$$

246 (b)

Let $y = \sqrt{\sec\sqrt{x}}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{\sec\sqrt{x}}} \cdot \sec\sqrt{x} \cdot \tan\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{4\sqrt{x}}(\sec\sqrt{x})^{3/2} \cdot \sin x$$

247 (c)

We have,

$$Sgn\, x = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$$

$$\therefore g(x) = Sgn\,\sin x = \begin{cases} \sin x, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -\sin x, & \text{for } x < 0 \end{cases}$$

$$\Rightarrow g'(x) = \begin{cases} \cos x, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -\cos x, & \text{for } x < 0 \end{cases}$$

$$\Rightarrow g'(1) = \cos 1$$

248 (c)

$$\because y = x^{\ln x}$$

On taking log on both sides, we get

$$\ln y = (\ln x)^2$$

On differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\ln x}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\frac{2\ln x}{x} = \frac{2(x^{\ln x})\ln x}{x}$$

$$\Rightarrow \frac{dy}{dx} = 2x^{\ln x - 1}\ln x$$

249 (d)

Since, $x = e^t\sin t$ and $y = e^t\cos t$

$$\Rightarrow \frac{dx}{dt} = e^t\cos t + \sin t\, e^t$$

and $\frac{dy}{dt} = -e^t\sin t + e^t\cos t$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{(\cos t - \sin t)}{(\cos t + \sin t)}$$

$$\frac{d^2y}{dx^2} = \frac{\left[\begin{array}{c}(\cos t + \sin t)(-\sin t - \cos t)\\ -(\cos t - \sin t)(-\sin t + \cos t)\end{array}\right]}{(\cos t + \sin t)^2}\frac{dt}{dx}$$

$$= \frac{-(\sin t + \cos t)^2 - (\cos t - \sin t)^2}{(\cos t + \sin t)^2}$$

$$\times \frac{1}{e^t(\cos t + \sin t)}$$

$$= -\frac{2}{e^t(\cos t + \sin t)^3}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{(x=\pi)} = \frac{-2}{e^\pi(\cos\pi + \sin\pi)^3} = \frac{2}{e^\pi}$$

250 (b)

We have,

$$f'(x) = \sin(\log x) \text{ and } y = f\left(\frac{2x+3}{3-2x}\right)$$

$$\therefore \frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right) \times \frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\} \times \frac{12}{(3-2x)^2} \quad [\because f'(x) = \sin(\log x)]$$

251 (d)

Given, $r = \left[2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)\right]^{1/2}$

$$\frac{dr}{d\phi} = \frac{\left[2 - 2\cos\left(2\phi + \frac{\pi}{4}\right)\sin\left(2\phi + \frac{\pi}{4}\right).2\right]}{2\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$$

$$= \frac{\left[1 - \sin\left(4\phi + \frac{\pi}{2}\right)\right]}{\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$$

$$\Rightarrow \left(\frac{dr}{d\phi}\right)_{\phi=\pi/4} = \frac{\left[1 - \sin\left(\pi + \frac{\pi}{2}\right)\right]}{\sqrt{2.\frac{\pi}{4} + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{4}\right)}}$$

$$= \frac{1 + 1}{\sqrt{\frac{\pi}{2} + \frac{1}{2}}} = 2\sqrt{\frac{2}{1 + \pi}}$$

252 (d)

$$\because y = 1 + x + x^2 + \ldots \infty$$

$$\therefore y = \frac{1}{1 - x} = (1 - x)^{-1}$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -\frac{1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}$$

$$\therefore \frac{dy}{dx} - y = \frac{1}{(1-x)^2} - \frac{1}{(1-x)}$$

$$= \frac{1 - 1 + x}{(1-x)^2} = \frac{x}{(1-x)^2}$$

$$\Rightarrow \frac{dy}{dx} - y = xy^2$$

$$\Rightarrow \frac{dy}{dx} = xy^2 + y$$

253 (a)

$$f'(5) = \lim_{h\to 0}\frac{f(5+h) - f(5)}{h}$$

$$= \lim_{h\to 0}\frac{2f(5)f(h) - f(5)}{h}$$

$$= \lim_{h\to 0} 2f(5)\left[\frac{f(h) - \frac{1}{2}}{h}\right]$$

$\Rightarrow 1024 \log 2 = 2f(5)f'(0)$

Again now, $f(2 + 3) = 2f(2)f(3)$ ... (i)

$\Rightarrow \dfrac{1024 \log 2}{2f'(0)} = 2 \times 8 \times f(3)$

$\Rightarrow f(3) = \dfrac{32 \log 2}{f'(0)}$ ...(ii)

$\therefore f'(3) = \lim\limits_{h \to 0} \log \dfrac{f(3+h)-f(3)}{h}$

$= \lim\limits_{h \to 0} \dfrac{2f(3)f(h) - f(3)}{h}$

$= 2f(3)f'(0)$

$= 2 \times \dfrac{32 \log 2 f'(0)}{f'(0)}$

$= 64 \log 2$ [from Eq. (ii)]

254 (d)

Let $u = e^{x^2}$ and $v = e^{2x-1}$

On differentiating w.r.t. x, we get

$\dfrac{du}{dx} = e^{x^2}.2x$ and $\dfrac{dv}{dx} = e^{2x-1}(2)$

$\therefore \dfrac{du}{dv} = \dfrac{e^{x^2}.2x}{e^{2x-1}.2}$

$\Rightarrow \dfrac{du}{dv} = xe^{x^2-2x+1}$

$\Rightarrow \left(\dfrac{du}{dv}\right)_{(x=1)} = 1.e^{1-2+1} = 1$

255 (b)

Given, $x = \log_e t$

$\Rightarrow e^x = t$ and $y + 1 = t^2 \Rightarrow y = e^{2x} - 1$

On differentiating w.r.t. y, we get

$2e^{2x}\dfrac{dx}{dy} = 1$

$\Rightarrow \dfrac{dx}{dy} = \dfrac{1}{2e^{2x}}$

Again, differentiating w.r.t. y, we get

$\dfrac{d^2x}{dy^2} = \dfrac{1}{2}e^{-2x}(-2)\dfrac{dx}{dy}$

$= -e^{-2x}.\dfrac{1}{2e^{2x}}$

$= -\dfrac{1}{2}e^{-4x}$

256 (a)

Given, $y = \cot^{-1}(\cos 2x)^{1/2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-1}{1 + \cos 2x} \times \dfrac{1}{2\sqrt{\cos 2x}} \times -2\sin 2x$

$= \dfrac{2\sin x \cos x}{2\cos^2 x\sqrt{\cos 2x}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\tan x}{\sqrt{\cos 2x}}$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{x=\frac{\pi}{6}} = \dfrac{1/\sqrt{3}}{\sqrt{1/2}} = \sqrt{\dfrac{2}{3}}$

257 (c)

Polynomial $P(x)$, satisfying the given relation can be taken as x

ie, $P(x) = x$

$\therefore P'(x) = 1$

$\Rightarrow P'(0) = 1$

258 (c)

$y = (\cos x^2)^2$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = 2\cos x^2 (-\sin x^2)2x = -2x \sin 2x^2$

259 (c)

We have,

$f(x) = 2^{2x-1}$ and $g(x) = -2^x + 2x \log 2$

$\therefore f'(x) > g'(x)$

$\Rightarrow 2 \times 2^{2x-1} \log 2 > -2^x \log 2 + 2 \log 2$

$\Rightarrow 2^{2x} > -2^x + 2$

$\Rightarrow 2^{2x} + 2^x - 2 > 0$

$\Rightarrow (2^x - 1)(2^x + 2) > 0$

$\Rightarrow 2^x - 1 > 0$

$\Rightarrow 2^x > 1 \Rightarrow x > 0 \Rightarrow x \in (0, \infty)$

260 (c)

Let $I = \dfrac{d}{dx}\left\{ \begin{array}{c} \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) + \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right) \\ - \tan^{-1}\left(\dfrac{4x-4x^3}{1-6x^2+x^4}\right) \end{array} \right\}$

Put $x = \tan\theta$ the given equation

$\therefore I = \dfrac{d}{dx}\{\tan^{-1}(\tan 2\theta)$

$+ \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 4\theta)\}$

$= \dfrac{d}{dx}(\theta) = \dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

261 (a)

$y = \dfrac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}$

$\times \dfrac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} + \sqrt{1+\sin x}}$

$= \dfrac{2(1+\cos x)}{-2\sin x} = -\cot\dfrac{x}{2}$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{2}\text{cosec}^2\dfrac{x}{2}$

262 (a)

Since, $y = x^{\sin x} + \sqrt{x}$

Let $y_1 = x^{\sin x}$ and $y_2 = \sqrt{x}$

Now, $y_1 = x^{\sin x} \Rightarrow \log y_1 = \sin x \log x$

On differentiating w.r.t. x, we get

$\dfrac{1}{y_1}.\dfrac{dy_1}{dx} = \cos x \log x + \dfrac{1}{x}\sin x$

$\Rightarrow \dfrac{dy_1}{dx} = x^{\sin x}[\cos x \log x + \dfrac{1}{x}\sin x]$

$\Rightarrow \left(\dfrac{dy_1}{dx}\right)_{x=\frac{\pi}{2}} = \left(\dfrac{\pi}{2}\right)^{\sin\frac{\pi}{2}}\left[\cos\dfrac{\pi}{2}\log\dfrac{\pi}{2} + \dfrac{2}{\pi}\sin\dfrac{\pi}{2}\right]$

$$= \frac{\pi}{2} \times \frac{2}{\pi} = 1$$

Now, $y_2 = \sqrt{x} \Rightarrow \frac{dy_2}{dx} = \frac{1}{2\sqrt{x}}$

$$\Rightarrow \left(\frac{dy_2}{dx}\right)_{x=\frac{\pi}{2}} = \frac{1}{2\sqrt{\frac{\pi}{2}}} = \sqrt{\frac{1}{2\pi}}$$

Since, $y = y_1 + y_2$

$\therefore$ At $x = \frac{\pi}{2}$, $\frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx} \Rightarrow \frac{dy}{dx}$

$$= 1 + \frac{1}{\sqrt{2\pi}}$$

263 (a)

Since, $y = \frac{3at^2}{1+t^2}$ and $x = \frac{3at}{1+t^3}$

On differentiating given curves w.r.t. t respectively

$$\frac{dy}{dt} = \frac{(1+t^3)(6at) - 3at^2(3t^2)}{(1+t^3)^2} = \frac{6at - 3at^4}{(1+t^3)^2}$$

and $\frac{dx}{dt} = \frac{(1+t^3)(3a) - 3at(3t^2)}{(1+t^3)^2} = \frac{3a - 6at^3}{(1+t^3)^2}$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3at(2-t^3)}{3a(1-2t^3)} = \frac{t(2-t^3)}{(1-2t^3)}$

264 (d)

Given, $y = \frac{\log x}{\log a} + \frac{\log a}{\log x} + 1 + 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x\log a} - \frac{\log a}{x(\log x)^2}$$

265 (c)

We have,

$$8f(x) + 6f\left(\frac{1}{x}\right) = x + 5 \quad ...(i)$$

Replacing x by $\frac{1}{x}$, we get

$$6f(x) + 8f\left(\frac{1}{x}\right) = \frac{1}{x} + 5 \quad ....(ii)$$

Eliminating $f\left(\frac{1}{x}\right)$ from these two equations, we get

$$f(x) = \frac{1}{28}\left(8x - \frac{6}{x} + 10\right)$$

$\therefore y = x^2 f(x) = \frac{1}{28}(8x^3 - 6x + 10x^2)$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{28}(24x^2 - 6 + 20x)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=-1} = \frac{1}{28}(24 - 6 - 20) = -\frac{1}{14}$$

266 (d)

Since, $y = \sqrt{\frac{1-x}{1+x}}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\sqrt{1+x} \times \frac{(-1)}{2\sqrt{1-x}} - \sqrt{1-x} \times \frac{1}{2\sqrt{1+x}}}{(\sqrt{1+x})^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{(1+x)\sqrt{1-x^2}} \times \frac{1-x}{1-x}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} + y = 0$$

267 (a)

$x^y = e^{2(x-y)}$

$\therefore y \log x = 2(x - y)$

$\Rightarrow y(\log x + 2) = 2x$

$$y = \frac{2x}{\log x + 2}$$

$$\frac{dy}{dx} = \frac{(\log x + 2)(2) - 2x \cdot \frac{1}{x}}{(\log x + 2)^2}$$

$$= \frac{2\log x + 4 - 2}{(\log x + 2)^2} = \frac{2(\log x + 1)}{(\log x + 2)^2}$$

268 (a)

$x = a(1 + \cos\theta), \quad y = a(\theta + \sin\theta)$

$$\frac{dx}{d\theta} = -a\sin\theta, \quad \frac{dy}{d\theta} = a(1 + \cos\theta)$$

$\therefore \frac{dy}{dx} = \frac{1+\cos\theta}{-\sin\theta} = \frac{2\cos^2\frac{\theta}{2}}{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$$\Rightarrow \frac{dy}{dx} = -\cot\frac{\theta}{2}$$

$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right) \cdot \frac{d\theta}{dx}$

$$= \frac{d}{d\theta}\left(-\cot\frac{\theta}{2}\right) \cdot \frac{1}{-a\sin\theta}$$

$$= \frac{1}{2}\text{cosec}^2\frac{\theta}{2} \cdot \frac{1}{-a\sin\theta}$$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}} = \frac{1}{2} \cdot 2 \cdot \frac{1}{-a} = -\frac{1}{a}$

269 (a)

Given, $y^2 = ax^2 + bx + c \Rightarrow 2y\frac{dy}{dx} = 2ax + b$

$$\Rightarrow 2\left(\frac{dy}{dx}\right)^2 + 2y\left(\frac{d^2y}{dx^2}\right) = 2a$$

$$\Rightarrow y\frac{d^2y}{dx^2} = a - \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow y\frac{d^2y}{dx^2} = a - \left(\frac{2ax+b}{2y}\right)^2$$

$$\Rightarrow y\frac{d^2y}{dx^2} = \frac{4ay^2 - (2ax+b)^2}{4y^2}$$

$$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 4a(ax^2 + bx + c) - (4a^2x^2 + 4abx + b^2)$$

$$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 4ac - b^2$$

$$\Rightarrow y^3\frac{d^2y}{dx^2} = \frac{4ac-b^2}{4} = \text{constant}$$

270 (d)

Given, $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \cdots = \frac{1}{1-\frac{1}{x}}$

$\Rightarrow \quad y = \dfrac{x}{x-1}$ (GP series) ...(i)

$\dfrac{dy}{dx} = \dfrac{1(x-1) - x.1}{(x-1)^2} = -\dfrac{1}{(x-1)^2}$

$\Rightarrow \quad \dfrac{dy}{dx} = -\dfrac{y^2}{x^2}$ [from Eq.(i)]

271 (c)

Given, $f(x, y) = 2(x-y)^2 - x^4 - y^4$

On differentiating partially w.r.t. x, twicely

$f_x = 4(x-y) - 4x^3$

$\Rightarrow f_{xx} = 4 - 12x^2$

$\Rightarrow (f_{xx})_{(0,0)} = 4 - 0 = 4$

Similarly, $f_{yy} = 4 - 12y^2$

$\Rightarrow (f_{yy})_{(0,0)} = 4 - 0 = 4$

and $f_{xy} = -4 + 0$

$\Rightarrow (f_{xy})_{(0,0)} = -4$

$\therefore \left| f_{xx} f_{yy} - f_{xy}^2 \right|_{(0,0)} = |4(4) - (-4)^2| = 0$

272 (c)

$y = \tan^{-1} \dfrac{1}{1 + x + x^2} + \tan^{-1} \dfrac{1}{x^2 + 3x + 3} +$

...+upto n terms

$= \tan^{-1} \dfrac{(x+1) - x}{1 + x(x+1)}$

$\qquad + \tan^{-1} \dfrac{(x+2) - (x+1)}{1 + (x+1)(x+2)} +$

...+ upto n terms

$= [\tan^{-1}(x+1) - \tan^{-1} x + \tan^{-1}(x+2)$

$\qquad - \tan^{-1}(x+1)$

$+ \cdots + \tan^{-1}(x+n) - \tan^{-1}\{x + (n-1)\}]$

$= \tan^{-1}(x+n) - \tan^{-1} x$

$\therefore y'(x) = \dfrac{1}{1 + (x+n)^2} - \dfrac{1}{1 + x^2}$

$\Rightarrow y'(0) = \dfrac{1}{1 + n^2} - 1 = \dfrac{n^2}{1 + n^2}$

273 (b)

Let $f(x) = x^2$

On differentiating w.r.t. x, we get

$f'(x) = 2x$

Given that, $f'(a+b) = f'(a) + f'(b)$

$\Rightarrow 2(a+b) = 2a + 2b$

$\Rightarrow 2a + 2b = 2a + 2b$

274 (c)

Let $y = (x+1)^n$

$\therefore \dfrac{dy}{dx} = n(x+1)^{n-1}$

$\dfrac{d^2 y}{dx^2} = n(n-1)(x+1)^{n-2}$

Similarly, $\dfrac{d^2 y}{dx^n} = n(n-1)(n-2) \ldots 3.2.1 = n!$

275 (d)

Given, $y = 2^x . 3^{2x-1}$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = 2^x . 3^{2x-1} \log 3 (2) + 2^x . 3^{2x-1} \log 2$

$\Rightarrow \dfrac{dy}{dx} = 2^x . 3^{2x-1} [2 \log 3 + \log 2]$

$\Rightarrow \dfrac{dy}{dx} = y \log 18$

276 (a)

Given, $y = x^2 + \dfrac{1}{y} \Rightarrow y^2 = x^2 y + 1$

$\Rightarrow 2y \dfrac{dy}{dx} = y.2x + x^2 \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{2xy}{2y - x^2}$

277 (d)

$y = \dfrac{\frac{(e^x + e^{-x})}{2}}{\frac{(e^x - e^{-x})}{2}} = \dfrac{\cosh x}{\sinh x}$

$\Rightarrow y = \coth x \Rightarrow \dfrac{dy}{dx} = -\text{cosech}^2 x$

278 (c)

$f(f(x)) = f(|x-2|)$

$= ||x - 2| - 2|$

$= x - 4 \quad (\because x > 20)$

$\Rightarrow g(x) = x - 4$

$\therefore g'(x) = 1$

279 (b)

$5f(x) + 3f\left(\dfrac{1}{x}\right) = x + 2$ ... (i)

On replacing x by $\dfrac{1}{x}$, we get

$5f\left(\dfrac{1}{x}\right) + 3f(x) = \dfrac{1}{x} + 2$ ... (ii)

On multiplying Eq. (i) by 5 and Eq. (ii) by 3 and then on subtracting, we get

$\therefore 16 f(x) = 5x - \dfrac{3}{x} + 4$

$\Rightarrow x f(x) = \dfrac{5x^2 - 3 + 4x}{16} = y$

$\therefore \dfrac{dy}{dx} = \dfrac{10x + 4}{16}$

$\dfrac{dy}{dx}\bigg|_{x=1} = \dfrac{10 + 4}{16} = \dfrac{7}{8}$

280 (b)

$f(\log x) = \log \log(x)$

$\Rightarrow \dfrac{d}{dx}\{f(\log x)\} = \dfrac{1}{x} . \dfrac{1}{\log x} = (x \log x)^{-1}$

282 (a)

Given, $f(x) = |x|^3 = \begin{cases} 0, & x = 0 \\ x^3, & x > 0 \\ -x^3, & x < 0 \end{cases}$

Now, $R f'(0) = \lim_{h \to 0} \dfrac{f(h) - f(0)}{h}$

$= \lim_{h \to 0} \dfrac{h^3 - 0}{h} = 0$

and $L f'(0) = \lim_{h \to 0} \dfrac{f(-h) - f(0)}{-h}$

$$= \lim_{h \to 0} \frac{-h^3 - 0}{-h} = 0$$
$$\because Rf'(0) = Lf'(0) = 0$$
$$\therefore f'(0) = 0$$

283 (c)

We have, $y = \log|x| = \begin{cases} \log x, & x > 0 \\ \log(-x), & x < 0 \end{cases}$

$$\therefore \frac{dy}{dx} = \begin{cases} \dfrac{1}{x}, & x > 0 \\ \dfrac{1}{-x}(-1) = \dfrac{1}{x}, & x < 0 \end{cases}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x}, x \neq 0$$

285 (b)

We have, $f(x) = x + 2$

$\therefore f'(x) = 1$ for all $x \Rightarrow f'(x) = 1$ for all x

286 (c)

We have,

$$f(x) - \log_x(\log_e x)$$

$$\Rightarrow f(x) = \frac{\log_e(\log_e x)}{\log_e x}$$

$$\Rightarrow f'(x) = \frac{\log_e x \times \frac{1}{x\log_e x} - \frac{1}{x}\log_e(\log_e x)}{(\log_e x)^2}$$

$$\Rightarrow f'(e) = \frac{1}{e} - \frac{1}{e} \times \log(1) = \frac{1}{e}$$

287 (b)

Given, $\sin(x + y) + \cos(x + y) = \log(x + y)$

On differentiating w.r.t. x, we get

$$\cos(x+y)\left(1 + \frac{dy}{dx}\right) - \sin(x+y)\left(1 + \frac{dy}{dx}\right)$$

$$= \frac{1}{(x+y)}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow 1 + \frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$$

288 (b)

$$10^{-x\tan x}\frac{d}{dx}(10^{x\tan x})$$

$$= 10^{-x\tan x}10^{x\tan x}\log 10(\tan x + x\sec^2 x)$$

$$= \log 10(\tan x + x\sec^2 x)$$

289 (c)

Given, $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \cdots$

$$\Rightarrow y = e^{-x} \Rightarrow \frac{dy}{dx} = -e^{-x}$$

$$\therefore \frac{d^2y}{dx^2} = e^{-x} = y$$

290 (b)

Let $y_1 = \sec^{-1}\frac{1}{2x^2-1}$ and $y_2 = \sqrt{1-x^2}$

$$\Rightarrow \frac{dy_1}{dx} = \frac{-2}{\sqrt{1-x^2}} \text{ and } \frac{dy_2}{dx} = \frac{-x}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy_1}{dy_2} = \frac{2}{x} \Rightarrow \left(\frac{dy_1}{dy_2}\right)_{x=1/2} = 4$$

291 (d)

We have,

$$y = \cos 2x \cos 3x = \frac{1}{2}[\cos 5x + \cos x]$$

$$\therefore y_n = \frac{1}{2}\left\{\frac{d^n}{dx^n}(\cos 5x) + \frac{d^n}{dx^n}(\cos x)\right\}$$

$$\Rightarrow y_n = \frac{1}{2}\left\{5^n \cos\left(\frac{n\pi}{2} + 5x\right) + \cos\left(\frac{n\pi}{2} + x\right)\right\}$$

292 (c)

Given, $f(x) = \log_{x^2}(\log_e x) = \frac{1}{2}\log_x(\log_e x)$

$$\Rightarrow (x) = \frac{1}{2}\frac{\log_e \log_e x}{\log_e x}$$

$$\Rightarrow f'(x) = \frac{1}{2}\frac{\log_e x\left(\frac{1}{x\log_e x}\right) - \log_e \log_e x \times \frac{1}{x}}{(\log_e x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2}\frac{\frac{1}{x} - \frac{1}{x}\log_e \log_e x}{(\log_e x)^2}$$

At $x = e$, $f'(e) = \frac{1}{2}\frac{\frac{1}{e} - \frac{1}{e}\log_e 1}{(1)^2}$

$$\Rightarrow f'(e) = \frac{1}{2e}$$

293 (d)

Given, $f(x) = \sin x, g(x) = x^2$

and $h(x) = \log_e x$

Also, $F(x) = (hogof)(x)$

$\therefore (hogof)(x) = (hog)(\sin x)$

$\Rightarrow \qquad = h(\sin x^2)$

$\Rightarrow F(x) = 2\log\sin x$

On differentiating, we get

$F'(x) = 2\cot x$

Again differentiating, we get

$F''(x) = -2\csc^2 x$

294 (c)

Given, $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

and

$$y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$$

$$\Rightarrow = \tan^{-1} t,$$

and $y = \tan^{-1} t$

$$\therefore y = x \Rightarrow \frac{dy}{dx} = 1$$

295 (c)

$$\because y = \tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log ex^2}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$$

$$= \tan^{-1}\left(\frac{1 - \log x^2}{1 + \log x^2}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$$

$$= \tan^{-1}(1) - \tan^{-1}(2\log x) + \tan^{-1}(3) + \tan^{-1}(2\log x)$$

$$\therefore y = \tan^{-1}(1) + \tan^{-1}(3)$$

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$$

296 (b)

Let $y = \sin^2 \cot^{-1} \left\{ \sqrt{\frac{1-x}{1+x}} \right\}$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$\Rightarrow y = \sin^2 \cot^{-1} \left\{ \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \right\}$

$\Rightarrow y = \sin^2 \cot^{-1} \left\{ \sqrt{\frac{2\sin^2 \frac{\theta}{2}}{2\cos^2 \frac{\theta}{2}}} \right\}$

$= \sin^2 \cot^{-1} \left(\tan\frac{\theta}{2} \right)$

$\Rightarrow y = \sin^3 \left(\frac{\pi}{2} - \frac{\theta}{2} \right)$

On differentiating w.r.t. θ, we get

$\frac{dy}{d\theta} = 2\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) \cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) - \left(-\frac{1}{2}\right)$

$\Rightarrow \frac{dy}{d\theta} = -\frac{\sin(\pi-\theta)}{2} = -\frac{\sin\theta}{2} = -\frac{1}{2}\sqrt{1-x^2}$

$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \frac{-1}{2}\sqrt{1-x^2} \frac{d}{dx}(\cos^{-1} x)$

$= -\frac{\sqrt{1-x^2}}{2} \left(\frac{-1}{\sqrt{1-x^2}} \right) = \frac{1}{2}$

297 (c)

In the neighbourhood of $x = \sqrt[5]{\frac{\pi}{2}}$, we have

$[x] = 1$

Therefore, in the neighbourhood of $x = \sqrt[5]{\frac{\pi}{2}}$, we have

$f(x) = \sin\left\{ \frac{\pi}{2}[x] - x^5 \right\} = \sin\left(\frac{\pi}{2} - x^5 \right) = \cos x^5$

$\Rightarrow f'(x) = -5x^4 \sin x^5$

$\Rightarrow f'\left(\sqrt[5]{\frac{\pi}{2}} \right) = -5\left(\frac{\pi}{2}\right)^{4/5} \sin\frac{\pi}{2} = -5\left(\frac{\pi}{2}\right)^{4/5}$

298 (b)

Since, $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

Now, $f'(x) = g(x)$ and $f''(x) = -f(x)$

$\Rightarrow f''(x) = g'(x)$ and $f''(x) = -f(x)$

$\Rightarrow -f(x) = g'(x)$

Thus, $f'(x) = g(x)$ and $g'(x) = -f(x)$

$\therefore h'(x) = -2g(x)\,g'(x) + 2g(x)g'(x)$

$= 0, \forall x$

$\Rightarrow h(x) =$ constant for all x

But $h(5) = 11$

Hence, $h(x) = 11$ for all x

299 (d)

$$f(x) = \begin{vmatrix} x^3 & x^4 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$\Rightarrow \frac{d}{dx} f(x) = \begin{vmatrix} 3x^2 & 4x^3 & 6x \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$

$\Rightarrow \frac{d^2}{dx^2} f(x) = \begin{vmatrix} 6x & 12x^2 & 6 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$

$\Rightarrow \frac{d^3}{dx^3} f(x) = \begin{vmatrix} 6 & 24x & 0 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$

$\Rightarrow \frac{d^4}{dx^4} f(x) = \begin{vmatrix} 0 & 24 & 0 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$

$= -24 \begin{vmatrix} 1 & 4 \\ p & p^3 \end{vmatrix} = -24(p^3 - 4p)$

Hence, $\frac{d^4}{dx^4} f(x)$ is a constant.

301 (a)

Here, $y = \sin\left[\log\left(\frac{2x+3}{3-2x} \right) \right]$

$\Rightarrow \frac{dy}{dx} = \cos\left[\log\left(\frac{2x+3}{3-2x} \right) \right] \cdot \frac{3-2x}{2x+3} \cdot \frac{12}{(3-2x)^2}$

$= \frac{12}{9-4x^2} \cdot \cos\left[\log\left(\frac{2x+3}{3-2x} \right) \right]$

302 (c)

$y = \cos^{-1}\left(\frac{1-\log x}{1+\log x} \right)$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = -\frac{1}{\sqrt{1 - \left(\frac{1-\log x}{1+\log x} \right)^2}}$

$\frac{(1+\log x)\left(\frac{-1}{x}\right) - (1-\log x)\left(\frac{1}{x}\right)}{(1+\log x)^2}$

$= -\frac{1}{2\sqrt{\log x}} \cdot \frac{-1-\log x - 1 + \log x}{x(1+\log x)}$

$= \frac{1}{x\sqrt{\log x}\,(1+\log x)}$

$\therefore \left(\frac{dy}{dx} \right)_{x=e} = \frac{1}{e(1+1)} = \frac{1}{2e}$

303 (a)

Since, $x^y = y^x \Rightarrow y \log x = x \log y$

On differentiating w.r.t. x, we get

$y \cdot \frac{1}{x} + \log x \frac{dy}{dx} = x \cdot \frac{1}{y}\frac{dy}{dx} + \log y$

$\Rightarrow x(x - y\log x)\frac{dy}{dx} = y(-x\log y + y)$

304 (b)

Since x is the integral function of $(1+9y^2)^{-1/2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1+9y^2}} \Rightarrow \frac{dy}{dx} = \sqrt{1+9y^2}$

305 (d)

Given, $y = \sec^{-1}(\csc x) + \csc^{-1}(\sec x) + \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

$$= \sec^{-1}\left[\sec\left(\frac{\pi}{2} - x\right)\right] + \text{cosec}^{-1}\left[\text{cosec}\left(\frac{\pi}{2} - x\right)\right]$$
$$+ \sin^{-1}\left[\sin\left(\frac{\pi}{2} - x\right)\right]$$
$$+ \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right]$$
$$= \frac{\pi}{2} - x + \frac{\pi}{2} - x + \frac{\pi}{2} - x + \frac{\pi}{2} - x$$
$$\Rightarrow y = 2\pi - 4x$$

On differentiating w.r.t. x, we get
$$\Rightarrow \frac{dy}{dx} = -4$$

306 (b)

Let $f(x) = |x - 1| + |x - 5|$
$$f(x) = \begin{cases} -(x-1) - (x-5), & x < 1 \\ (x-1) - (x-5), & 1 \le x \le 5 \\ x - 1 + x - 5, & x > 5 \end{cases}$$
$$f(x) = \begin{cases} 6 - 2x, & x < 1 \\ 4, & x \le 5 \\ 2x - 6, & x > 5 \end{cases}$$
$\because x = 3 \in (1, 5)$
$\therefore$ For $x = 3$,
$$f(x) = 4 \Rightarrow f'(x) = 0$$

307 (a)

We have,
$$f(x) = \log_e\left\{\frac{u(x)}{v(x)}\right\}$$
$$\Rightarrow f'(x) = \frac{v(x)}{u(x)}\left\{\frac{v(x)u'(x) - u(x)v'(x)}{\{v(x)\}^2}\right\} \text{ for all } x$$
$$\Rightarrow f'(1) = 0$$

308 (b)

We have,
$$y = \sin^2\left\{\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right\} = \frac{1}{\text{cosec}^2\left\{\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right\}}$$
$$\Rightarrow y = \frac{1}{1 + \cot^2\left\{\cot^2\sqrt{\frac{1-x}{1+x}}\right\}} = \frac{\frac{1}{1}}{1+x} = \frac{1+x}{2}$$
$$\therefore \frac{dy}{dx} = \frac{1}{2}$$

309 (c)

$\because x = \dfrac{1 - t^2}{1 + t^2}$ and $y = \dfrac{2t}{1 + t^2}$

Put $t = \tan\theta$ in both the equations, we get
$$x = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \cos 2\theta \ldots \text{(i)}$$
and $y = \dfrac{2\tan\theta}{1 + \tan^2\theta} = \sin 2\theta \ldots$ (ii)

On differentiating both the Eqs.(i) and (ii), we get
$$\frac{dx}{d\theta} = -2\sin 2\theta \text{ and } \frac{dy}{d\theta} = 2\cos 2\theta$$

Therefore, $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\dfrac{\cos 2\theta}{\sin 2\theta} = -\dfrac{x}{y}$

310 (b)

Given, $x = y\sqrt{1 - y^2}$
$$\Rightarrow 1 = \frac{dy}{dx}\sqrt{1 - y^2} + \frac{y(-y)}{\sqrt{1 - y^2}}\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}\left[\frac{1 - y^2 - y^2}{\sqrt{1 - y^2}}\right] = 1 \Rightarrow \frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{1 - 2y^2}$$

311 (b)

We have,
$$f(x) = (x + 1)\tan^{-1}(e^{-2x})$$
$$f'(x) = \tan^{-1}(e^{-2x}) + \frac{(x+1)(-2)}{1 + e^{-4x}}e^{-2x}$$
$$\Rightarrow f'(0) = \tan^{-1}(1) - \frac{2}{2} = \frac{\pi}{4} - 1$$

312 (d)

Given, $y = 5^x x^5 \Rightarrow \dfrac{dy}{dx} = 5^x \log 5 . x^5 + 5^x 5x^4$
$$= 5^x(x^5 \log 5 + 5x^4)$$

313 (b)

$$y = a\cos(\log x) + b\sin(\log x)$$
On differentiating w.r.t. x, we get
$$y' = \frac{-a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$$
$$\Rightarrow xy' = -a\sin(\log x) + b\cos(\log x)$$
Again, on differentiating w.r.t. x, we get
$$xy'' + y' = -a\cos(\log x)\frac{1}{x} - b\sin(\log x)\frac{1}{x}$$
$$\Rightarrow x^2y'' + y'x = -[a\cos(\log x) + b\sin(\log x)]$$
$$\Rightarrow x^2y'' + xy' = -y$$

314 (d)

We have,
$$y = \sin^{-1}\left(\frac{\sin\alpha\sin x}{1 - \cos\alpha\sin x}\right)$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1 - \frac{\sin^2\alpha\sin^2 x}{(1 - \cos\alpha\sin x)^2}}}\frac{d}{dx}\left(\frac{\sin\alpha\sin x}{1 - \cos\alpha\sin x}\right)$$
$$\Rightarrow \frac{dy}{dx} = \frac{(1 - \cos\alpha\sin x)}{\sqrt{1 + \sin^2 x - 2\cos\alpha\sin x}}$$
$$\cdot \frac{\sin\alpha}{\cos\alpha}\frac{d}{dx}\left(1 - \frac{1}{1 - \cos\alpha\sin x}\right)$$
$$\Rightarrow \frac{dy}{dx} = -\frac{\tan\alpha(1 - \cos\alpha\sin x)}{\sqrt{1 + \sin^2 x - 2\cos\alpha\sin x}}\frac{d}{dx}\left(1\right.$$
$$\left. - \frac{1}{1 - \cos\alpha\sin x}\right)$$
$$\Rightarrow \frac{dy}{dx}$$
$$= \frac{\tan\alpha(1 - \cos\alpha\sin x)}{\sqrt{1 + \sin^2 x - 2\sin x\cos\alpha}}\left\{\frac{1 - \cos\alpha\cos x}{(1 - \cos\alpha\sin x)^2}\right\}$$

$$\Rightarrow \frac{dy}{dx}$$

$$= \frac{\sin\alpha \cos x}{(1 - \cos\alpha \sin x)\sqrt{1 + \sin^2 x - 2\cos\alpha \sin x}}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \sin\alpha \Rightarrow y'(0) = \sin\alpha$$

315 (a)

For $x < -1$

$f(x) = -(x-2) - (x+1) - x = -3x + 1$

$\therefore \qquad f'(x) = -3$

$\Rightarrow \qquad f'(-10) = -3$

316 (c)

Since $f(x)$ is an even function

$\therefore f(-x) = f(x)$

$\Rightarrow -f'(-x) = f'(x)$

$\Rightarrow f''(-x) = f''(x) \Rightarrow f''(-\pi) = f''(\pi) = 1$

317 (d)

$$\frac{d}{dx}[\log_e e^{\sin(x^2)}] = \frac{d}{dx}[\sin(x^2)] = \cos^{(x^2)} 2x$$

318 (c)

$$\frac{d}{dx}[f^2(x) + g^2(x)]$$

$$= 2[f(x)f'(x) + g(x)g'(x)]$$

$$= 2[f(x)g(x) + g(x)\{-f(x)\}]$$

$$= 0$$

$\Rightarrow f^2(x) + g^2(x) = \text{constant}$

$\therefore f^2(4) + g^2(4) = f^2(2) + g^2(2)$

$$= (4)^2 + (4)^2$$

$$= 32$$

319 (a)

Given, $z = \tan(y + ax) + \sqrt{y - ax}$

$$\Rightarrow z_x = \sec^2(y + ax)a + \frac{1}{2\sqrt{y - ax}}(-a)$$

$$\Rightarrow z_{xx} = 2\sec^2(y + ax)\tan(y + ax)a^2$$

$$+ \frac{1(-a^2)}{4(y - ax)^{3/2}}$$

and $z_y = \sec^2(y + ax) + \frac{1}{2\sqrt{y-ax}}$

$$\Rightarrow z_{yy} = 2\sec^2(y + ax)\tan(y + ax)$$

$$- \frac{1}{4(y - ax)^{3/2}}$$

$\therefore z_{xx} - a^2 z_{yy} = 0$

320 (c)

$$\frac{d}{dx}(\log x)^4 = 4(\log x)^3 \frac{d}{dx}(\log x) = \frac{4(\log x)^3}{x}$$

321 (d)

We have,

$$\sin^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \log a$$

$$\Rightarrow \frac{x^2 - y^2}{x^2 + y^2} = \sin(\log a) = \lambda \text{ (say)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{\lambda + 1}{1 - \lambda}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{x^2}{y^2}\right) = 0$$

$$\Rightarrow \frac{2xy^2 - 2x^2 y \frac{dy}{dx}}{y^4} = 0 \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

323 (b)

We have, $f(x) = \sqrt{1 + \cos^2(x^2)}$ ... (i)

On differentiating Eq. (i) w.r.t. x, we get

$$f'(x) = \frac{-2\sin x^2 \cos x^2}{\sqrt{1 + \cos^2 x^2}}(x)$$

$$\Rightarrow f'(x) = \frac{-\sin 2x^2}{\sqrt{1 + \cos^2 x^2}}(x) \text{ ... (ii)}$$

Put, $x = \frac{\sqrt{\pi}}{2}$ in Eq. (ii), we get

$$f'\left(\frac{\sqrt{\pi}}{2}\right) = -\frac{\sqrt{\pi}}{2} \cdot \frac{\sin 2\left(\frac{\pi}{4}\right)}{\sqrt{1 + \frac{1}{2}}}$$

$$= -\frac{\sqrt{\pi}}{2} \cdot \frac{\sin \frac{\pi}{2}}{\sqrt{\frac{3}{2}}} = -\sqrt{\frac{\pi}{6}}$$

324 (d)

Let $y = \log x$

On differentiating w.r.t. x from 1 to n times, we get

$$y_1 = \frac{1}{x}, y_2 = \frac{-1}{x^2}, y_3 = \frac{2}{x^3} \cdots$$

$$y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$$

325 (d)

Since, $\frac{dx}{dy} = \frac{1}{dy/dx} = \left(\frac{dy}{dx}\right)^{-1}$

$$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}\frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}\left(\frac{dx}{dy}\right) = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$$

326 (c)

Let $y = a\sin^3 t, \ x = a\cos^3 t$

On differentiating w.r.t. t, we get

$$\frac{dy}{dt} = 3a\sin^2 t \cos t, \frac{dx}{dt} = -3a\cos^2 t \sin t$$

$$\therefore \frac{dy}{dx} = \frac{3a\sin^2 t \cos t}{-3a\cos^2 t \sin t}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sin t}{\cot t} = -\tan t$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -\sec^2 t \cdot \frac{dt}{dx}$$

$$= -\frac{\sec^2 t}{-3a\cos^2 t\, \sin t}$$

$$= \frac{1}{3a\cos^4 t \sin t}$$

$$\Rightarrow \left(\frac{d^2 y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3a\left(\frac{1}{\sqrt{2}}\right)^4 \cdot \left(\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{(\sqrt{2})^5}{3a} = \frac{4\sqrt{2}}{3a}$$

327 (a)

$$\frac{dy}{dx} = \left(\frac{dx}{dy}\right)^{-1}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = -\left(\frac{dx}{dy}\right)^{-2}\left\{\frac{d}{dx}\left(\frac{dx}{dy}\right)\right\}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = -\left(\frac{dx}{dy}\right)^{-2}\left\{\frac{d}{dy}\left(\frac{dx}{dy}\right)\frac{dy}{dx}\right\}$$

$$= -\left(\frac{dy}{dx}\right)^2\left\{\frac{d^2 x}{dy^2}\cdot\frac{dy}{dx}\right\} = -\left(\frac{dy}{dx}\right)^3\frac{d^2 x}{dy^2}$$

$$\Rightarrow \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^3\frac{d^2 x}{dy^2} = 0$$

328 (b)

We have,

$$e^y + xy = e$$

$$\Rightarrow e^y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0 \quad \text{...(i)}$$

$$\Rightarrow e^y\frac{d^2 y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} + x\frac{d^2 y}{dx^2} = 0 \quad \text{...(ii)}$$

Putting $x=0$ in $e^y + xy = e$, we get $y=1$

Putting $x=0, y=1$ in (i), we get

$$e\frac{dy}{dx} + 1 = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{e}$$

Putting $x=0, y=1, \frac{dy}{dx} = -\frac{1}{e}$ in (ii), we get

$$e\frac{d^2 y}{dx^2} + e\times\frac{1}{e^2} - \frac{2}{e} + 0\times\frac{d^2 y}{dx^2} = 0 \Rightarrow \frac{d^2 y}{dx^2} = \frac{1}{e^2}$$

329 (c)

Since, $\sin^{-1}\left(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right) = \sin^{-1} x + \sin^{-1}\sqrt{x}$

$$\therefore \frac{d}{dx}\{\sin^{-1}(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2})\}$$

$$= \frac{d}{dx}(\sin^{-1} x + \sin^{-1}\sqrt{x})$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

330 (d)

$$\because f(x) = \log_x(\log_e x) = \frac{\log_e\log_e x}{\log_e x}$$

$$\therefore f(x) = \frac{\left[\log_e x\frac{1}{\log_e x}\frac{1}{x} - \log\log_e x.\frac{1}{x}\right]}{(\log_e x)^2}$$

$$= \frac{1 - \log_e\log_e x}{x(\log_e x)^2}$$

$$\Rightarrow f'(e) = \frac{1 - \log_e\log_e e}{e(\log_e e)^2} = \frac{1 - \log_e 1}{e} = \frac{1}{e}$$

331 (a)

In the neighbourhood of $x = -\frac{\pi}{6}$, we observe that

$\cos 4x < 0$ and $\sin x < 0$

$$\Rightarrow |\cos 4x| = -\cos 4x \text{ and } |\sin x| = -\sin x$$

$$\therefore y = \log_{\sec 2x}(-\cos 4x) - \sin x$$

$$\Rightarrow y = \frac{\log_e(-\cos 4x)}{\log_e\sec 2x} - \sin x$$

$$\Rightarrow \frac{dy}{dx}\frac{(-4\tan 4x)\log_e\sec 2x - \log_e(-\cos 4x)\,2\tan}{\{\log_e\sec 2x\}^2}$$

$$- \cos x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=-\pi/6} = \left(\frac{-6\sqrt{3}}{\log_e 2} - \frac{\sqrt{3}}{2}\right)$$

332 (b)

Let $x^3 = \sin A$ and $y^3 = \sin B$. Then

$$\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$$

$$\Rightarrow \cos A + \cos B = a(\sin A - \sin B) \Rightarrow \cot\frac{A-B}{2}$$

$$= a$$

$$\Rightarrow A - B = 2\cot^{-1} a \Rightarrow \sin^{-1} x^3 - \sin y^3$$

$$= 2\cot^{-1} a$$

Differentiating w.r.t. to x

$$\frac{3x^2}{\sqrt{1-x^6}} - \frac{3y^2}{\sqrt{1-y^6}} = 0 \Rightarrow \frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$$

But, $\frac{dy}{dx} = f(x,y)\sqrt{\frac{1-y^6}{1-x^6}}$

$$\therefore f(x,y) = \frac{x^2}{y^2}$$

333 (d)

We have, $y = \log^n x$

Where $\log^n$ means log log ... (repeated n times).

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{x\log x\log^2 x \dots \log^{n-1} x}$$

$$\therefore x\log x\log^2 x \dots \log^{n-1} x\log^n x\frac{dy}{dx}$$

$$= \frac{x\log x\log^2 x \dots \log^{n-1} x\log^n x}{x\log x\log^2 x \dots \log^{n-1} x} = \log^n x$$

334 (b)

We have, $x^4 + y^4 = t^2 + \frac{1}{t^2}$

$$= \left(t + \frac{1}{t}\right)^2 - 2$$

$$= (x^2 + y^2)^2 - 2$$

$$= x^4 + y^4 + 2x^2 y^2 - 2$$

$$\Rightarrow x^2 y^2 = 1$$

On differentiating w.r.t. x, we get

$$x^2.2y\frac{dy}{dx} + y^2.2x = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

335 (b)

Given, $y = \sqrt{x \log_e x}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x \log_e x}}\left(x \times \frac{1}{x} + 1 \times \log_e x\right)$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=e} = \frac{1}{2\sqrt{e \times 1}}(1+1)$$

$$= \frac{1}{\sqrt{e}}$$

337 (c)

$$\frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right)\frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$$

$$= \sin\log\left(\frac{2x+3}{3-2x}\right)\left(\frac{(3-2x)(2)-(2x+3)(-2)}{(3-2x)^2}\right)$$

$$= \frac{12}{(3-2x)^2}\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=1} = \frac{12}{(3-2)^2}\cdot\sin\log 5 = 12\sin\log 5$$

338 (a)

Given, $x^y = e^{x-y}$

Taking log on both sides, we get

$y \log x = x - y$

On differentiating w.r.t. x, we get

$$\frac{y}{x} + \log x\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(1 + \log x) = 1 - \frac{y}{x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)}(1 + \log 1) = 1 - \frac{1}{1}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 0$$

339 (b)

Given, $y = \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$

Put $x = \sin\theta$

$$\therefore \quad y = \sin^{-1}\left(\frac{5\sin\theta + 12\cos\theta}{13}\right)$$

$$= \sin^{-1}\{\sin(\theta+\alpha)\}\left(\text{Put}\cos\alpha = \frac{5}{13}\right)$$

$$\Rightarrow y = \theta + \alpha = \sin^{-1}x + \alpha$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

340 (b)

Given, $y = e^x \cdot e^{x^2} \cdot e^{x^3} \dots e^{x^n} \dots$

$$\Rightarrow \quad y = e^{(x+x^2+\dots+\infty)}$$

$$\Rightarrow \quad y = e^{\frac{x}{1-x}}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{\frac{x}{1-x}}\left[\frac{(1-x)1-x(-1)}{(1-x^2)}\right]$$

$$= e^{\frac{x}{1-x}}\cdot\frac{1}{(1-x)^2}$$

At $x = \frac{1}{2}$,

$$\left(\frac{dy}{dx}\right)_{x=\frac{1}{2}} = e^{\frac{1/2}{1-1/2}}\cdot\frac{1}{\left(1-\frac{1}{2}\right)^2} = 4e$$

341 (a)

We have,

$$y = \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$$

$$\Rightarrow y = \sin^{-1}\left(\frac{5}{13}x + \frac{12}{13}\sqrt{1-x^2}\right)$$

$$\Rightarrow y = \sin^{-1}\left\{x\sqrt{1-\left(\frac{12}{13}\right)^2} + \frac{12}{13}\sqrt{1-x^2}\right\}$$

$$\Rightarrow y = \sin^{-1}x + \sin^{-1}\frac{12}{13} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

342 (a)

$$\frac{d}{dx} = \left(\tan^{-1}\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}}\right) = \frac{d}{dx}\tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{4}}{2\sin^2\frac{x}{4}}}$$

$$= \frac{d}{dx}\tan^{-1}\left|\cot\left(\frac{x}{4}\right)\right|$$

$$= \frac{d}{dx}\tan^{-1}\left|\tan\left(\frac{\pi}{2}-\frac{x}{4}\right)\right|$$

$$= \frac{d}{dx}\left(\frac{\pi}{2}-\frac{x}{4}\right) = \frac{-1}{4}$$

343 (b)

Let $p = \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

and $q = \cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1}x$

$$\therefore \quad p = q \quad \Rightarrow \quad \frac{dp}{dq} = 1$$

344 (a)

Let $f(x) = ax^2 + bx + c$

$\because \quad f(1) = f(-1)$

$\Rightarrow a+b+c = a-b+c \Rightarrow b = 0$

$\therefore \quad f(x) = ax^2 + c \quad \Rightarrow \quad f'(x) = 2ax$

$\therefore \quad f'(a_1) = 2aa_1, \quad f'(a_2) = 2aa_2,$

$f'(a_3) = 2aa_3$

Now assume

$2f'(a_2) = f'(a_1) + f'(a_3)$

$\Rightarrow 2.2aa_2 = 2aa_1 + 2aa_3$

$\Rightarrow 2a_2 = a_1 + a_3$

$\Rightarrow a_1, a_2, a_3$ are in AP.

$\therefore f'(a_1), f'(a_2), f'(a_3)$ are in AP.

345 (c)

Let $y_1 = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

and $y_2 = \cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\Rightarrow y_1 = 2\tan^{-1}x$ and $y_2 = 3\tan^{-1}x$

On differentiating w.r.t. x, we get

$\frac{dy_1}{dx} = \frac{2}{1+x^2}$ and $\frac{dy_2}{dx} = \frac{3}{1+x^2}$

$\Rightarrow \frac{dy_1}{dy_2} = \frac{\left(\frac{dy_1}{dx}\right)}{\left(\frac{dy_2}{dx}\right)} = \frac{\left(\frac{2}{1+x^2}\right)}{\left(\frac{3}{1+x^2}\right)} = \frac{2}{3}$

346 (b)

We have, $y = \log_{\sin x}\cos x$

$= \frac{\log\cos x}{\log\sin x}$

$= \frac{\log\sin x\frac{1}{\cos x}(-\sin x) - \log\cos x\frac{1}{\sin x}\cos x}{(\log\sin x)^2}$

$= \frac{-\tan x\log\sin x - \cot x\log\cos x}{(\log\sin x)^2}$

347 (c)

We have,

$y = \tan^{-1}\left(\frac{\log e - \log x^2}{\log e + \log x^2}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$

$\Rightarrow y = \tan^{-1}1 - \tan^{-1}(2\log x) + \tan^{-1}3$
$\qquad\qquad + \tan^{-1}(2\log x)$

$\Rightarrow y = \tan^{-1}1 + \tan^{-1}3$

$\Rightarrow \frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$

348 (b)

Differentiating $y = c\, e^{\frac{x}{x-a}}$ w.r.t. x, we get

$\frac{dy}{dx} = c\, e^{\frac{x}{x-a}} \cdot \frac{(-a)}{(x-a)^2} = -\frac{ay}{(x-a)^2}$

349 (d)

$y = \sin^{-1}\sqrt{1-x} \Rightarrow \frac{dy}{dx}$

$\qquad\qquad = \frac{1}{\sqrt{1-(1-x)}} \cdot \frac{1}{2\sqrt{1-x}} \cdot (-1)$

$= \frac{-1}{2\sqrt{x}\sqrt{1-x}}$

350 (d)

We have,

$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}} \times \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = -\frac{1+x^2}{\sqrt{(1-x^2)^2}} \times \frac{2(1+x^2) - 4x^2}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = -2\frac{(1+x^2)}{|1-x^2|} \times \frac{1-x^2}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = -2\left(\frac{1-x^2}{|1-x^2|}\right)\left(\frac{1}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \begin{cases} \frac{2}{1+x^2}, & \text{if } |x| > 1 \\ \frac{-2}{1+x^2}, & \text{if } |x| < 1 \end{cases}$

351 (b)

$y = ae^x + be^{-x} + c$

On differentiating w.r.t. x, we get

$y' = ae^x - be^{-x}$

Again differentiating w.r.t. x, we get

$y'' = ae^x + be^{-x}$

Again differentiating w.r.t. x, we get

$y''' = ae^x - be^{-x} = y'$